Exercise 1 – Gravimetry anomaly due to a sphere

As a first step to understand the interpretation of gravimetry anomalies, we could compute the theoretical response of a cave (Hohlraum) in the subsurface which is completely filled with water. For simplicity, let's assume that the cave can be described as a sphere, with a radius of 0.5 m and it is located 15 m below the surface, as depicted in Figure 1. Please, use matlab to plot expected gravity anomaly considering surface readings collected every 1m along the profile direction (x-direction in Figure 1), in a profile with a length of 200m with the center of the anomaly placed at the middle of the profile.

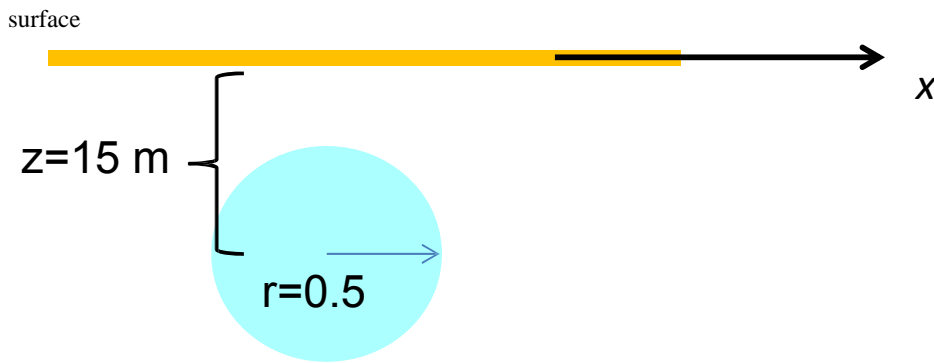The surrounding rocks are Shales (density ~2400 kg/m^3).

surface

z=15 m

r=0.5

Figure 1

The analytical equation to compute the vertical component of the gravity anomaly (Δg) associated with a sphere of constant density is given by:

$$\Delta g(x) = G \frac{\Delta M}{z^2 + x^2} \frac{z}{(z^2 + x^2)^{1/2}} \qquad (1)$$

where, ΔM is the difference in the mass from the target (sphere) and the surrounding material; z the distance (depth) between the center of the target (i.e., sphere) and the surface; x is the distance along profile direction, and a is the radius of the sphere.

Steps to solve the exercise:

1. Compute the mass difference between the subsurface and the target, i.e., the water sphere (Hint: check the difference in the density and the volume of the anomaly).
2. Implement in matlab the analytical equation (1)
3. Plot the vertical component of the gravimetry anomaly due to the sphere represented in Figure 1, for measurements performed in the surface (every 1 m along profile direction, in a profile with a length of 200m with the center of the anomaly placed at the middle of the profile).
4. Plot the gravitational effect due to the same sphere but for the center of the sphere located at a depth of 40 m (z=40) and at a depth of 3m.
5. Compute the analytical response of a sphere as illustrated in Figure 1, but assuming that it is filled with gold instead of water. The density of gold is 19300 (kg/m3). Plot the curve as obtained for the sphere filled with water and filled with gold. Discuss the observed response.
6. Repeat the step 3 for measurements performed every 20m – discuss the importance on the sampling (i.e., the number of measurements and the separation between them)

Note: gravimetric anomalies should be provided in miliGals – please check the units!

The work of the geophysicist is to define the geometry of the materials in the subsurface based on surface measurements. Hence, now let's try to define the depth at which the sphere is located only using the plots performed in previous steps

7. Plot a line (at the y-axis) to indicate the half of the maximum gravitational effect (Δg /2)
8. Find the distance along profile direction (x), at which the $\Delta g/2$ is located. This value is commonly referred to as the half width ($x_{1/2}$)
9. Use the analytical approximation (z = 1.3 $x_{1/2}$) to compute the depth of a sphere (from the surface to the center of the sphere).
10.   If the computed depth (Step 9) is not the same as the original value, discuss possible reasons to explain the inaccuracies.
11.   Compute the analytical gravity curve for a sphere filled with gold (similar to step 5) but placed at a depth of 70 m and for the case of a radius of 2m and, as well as for a gold sphere at 3m and a radius of 0.25m. Discuss the results.

12.   Could you define 2 spheres, with completely different characteristics (density, radius and depth) that provide approximately the same response?

Protocol has to be delivered as a .pdf file on October 21.2016 by 10:00 am to the address:

Hint: If you have a curve **D** as a function of **x**, i.e., **D**(**x**), and you want to find in the value of **x** at which a given value of **D** is given, then you can use in matlab the following script:

X=-200:0.5:200;

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idx = find(diff(D >= yval));
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xval = x(idx) + (yval - D(idx)) .* (x(idx+1) -
x(idx)) ./ (D(idx+1) - D(idx));
```

yval - is the value of function **D** that you know

idx – the index pointing to the *yval* in the variable representing the *x* vector

xval – the unknown *x*-value (at the *x*-coordinate) at which the known *yval* is found