

No homework for November 6; note that the lecture is back in Sem 136 (FH, 10th floor).

Exercise 4 (please return by November 13): The quaternions are the non-commutative generalization of the complex numbers given by $\mathbb{H} = \{q = x + iy + ju + kv : x, y, u, v \in \mathbb{R}\}$ with $i^2 = j^2 = k^2 = -1$, $ij = -ji = k$, $jk = -kj = i$, $ki = -ik = j$. Denote by $Sp(n)$ the group of linear transformations of \mathbb{H}^n that preserve the hermitian form $\sum_{i=1}^n \bar{q}_i r_i$, where $\bar{q} = x - iy - ju - kv$. Show that $Sp(n) = Sp(2n, \mathbb{C}) \cap U(2n)$.

Hint: Identify quaternions with pairs (z, w) of complex numbers via $q = z + jw$.