9. Tutorium - Resultate **03.06.2016**

9.1 Geteilter Kreiszylinder

a)
$V(r,\phi) = \frac{2V_0}{\pi} \cdot \begin{cases} \arctan\frac{2ar\sin\phi}{a^2-r^2} & \text{für } r \leq a, \\ \arctan\frac{2ar\sin\phi}{r^2-a^2} & \text{für } r \geq a. \end{cases}$
b) $\sigma(\phi) = \frac{4\epsilon_0 V_0}{a\pi}\frac{1}{\sin\phi}$.
c) Für $0 < \phi < \pi$: $\tau_1 \approx \frac{8\epsilon_0 V_0}{\pi}\log\frac{4a}{d}$.
Für $\pi < \phi < 2\pi$: $\tau_2 \approx -\frac{8\epsilon_0 V_0}{\pi}\log\frac{4a}{d}$.
Kapazität pro Längeneinheit: $C = \frac{4\epsilon_0}{\pi}\log\frac{4a}{d}$.

9.2 Legendrepolynome

a) $P_0 = 1$.
$P_1 = x$.
$P_2 = \frac{1}{2}\left(3x^2-1\right)$.
$P_3 = \frac{1}{2}\left(5x^3-3x\right)$.
b) $g(x,z) = 1 + xz + \frac{1}{2}(3x^2-1)z^2 + \frac{1}{2}(5x^3-3x)z^3$
c) Behandle $m \neq n$:
$\int_{-1}^{1} P_m(x)P_n(x)dx = \ldots = \frac{(-1)^n}{2^{m+n}m!n!}\int_{-1}^{1} dx(x^2-1)^n\frac{d^{m+n}}{dx^{m+n}}(x^2-1)^m = \ldots = 0 \qquad m < n$
Für $m = n$ gilt:
$\int_{-1}^{1} P_n(x)P_n(x)dx = \ldots = \frac{(-1)^n(2n)!}{(n!)^2 2^{2n}}\frac{(-1)^n(n!)^2 2^{1+2n}}{(2n+1)!} = \frac{2}{2n+1}$

9.3 Multipolmomente eines Ellipsoids

$q_{11} = q_{22} = q_{21} = q_{10} = 0$.
$q_{00} = \sqrt{\frac{1}{4\pi}}\frac{4\pi}{3}a^2c\rho_0$.
$q_{20} = \sqrt{\frac{5}{16\pi}}\frac{4\pi}{3}a^2c\rho_0\,\frac{2}{5}\left(c^2-a^2\right)$.
$V(x^m) = \frac{1}{4\pi\epsilon_0}\frac{4\pi}{3}a^2c\rho_0\left(\frac{1}{r} + \frac{c^2-a^2}{10}\,\frac{3\cos^2\vartheta-1}{r^3}\right)$.