

4. Tutorium VU Quantentheorie I, 05.11.2010 – Lösungen

Beispiel 1

a

$$\Psi_E(x) = \theta(-x) \left[e^{ikx} + \frac{iD}{k - iD} e^{-ikx} \right] + \theta(x) \left[\frac{k}{k - iD} e^{ikx} \right], \quad k = \sqrt{\frac{2mE}{\hbar^2}}$$

b

$$j_I(0) = j_{II}(0), \text{ da : } j(x) = j = \frac{\hbar k}{m} \frac{k^2}{k^2 + D^2}$$

c

$$j_{\text{ein}} = \frac{\hbar k}{m}, \quad j_{\text{ref}} = \frac{D^2}{k^2 + D^2} \frac{\hbar k}{m}, \quad j_{\text{trans}} = j_I = j_{II} = \frac{k^2}{k^2 + D^2} \frac{\hbar k}{m}$$

$$\Rightarrow j_{\text{ref}} + j_{\text{trans}} = j_{\text{ein}}$$

d

$$T = \frac{k^2}{k^2 + D^2}$$

$$|D| \ll k \Rightarrow T \rightarrow 1 - \frac{D^2}{k^2} \approx 1$$

$$|D| \gg k \Rightarrow T \rightarrow \frac{k^2}{D^2} \ll 1$$

$$\lim_{D \rightarrow \infty} T(D; k) = 0$$

e

$$\Psi_E(x) = \theta(-x) \left[e^{ikx} + \frac{iD}{k - iD} e^{-ikx} \right] + \theta(x) \left[\frac{k}{k - iD} e^{ikx} \right] = e^{ikx} + \frac{iD}{k - iD} e^{ik|x|}$$

$$k_{\text{Pol}} = iD \Rightarrow E_{\text{geb}} = -\frac{\hbar^2 D^2}{2m} = \frac{\hbar^2 k_{\text{Pol}}^2}{2m}$$

Beispiel 2

b

$$T = \frac{1}{5 - 4 \sin 4Da}$$

c

$$R = 1 - T$$

d

$$T_{\max} = 1 \quad \text{wenn} \quad 4Da = \frac{\pi}{2} + 2n\pi, \quad n \in \mathbb{N}$$

$$T_{\min} = \frac{1}{9} \quad \text{wenn} \quad 4Da = \frac{3\pi}{2} + 2n\pi, \quad n \in \mathbb{N}$$