(5) $H\psi = \left(-\frac{\hbar^2}{2m}\partial_x\partial_x + V(x)\right)\psi$

$V(x) = \begin{cases} 0 & 0 \le x \le L \\ \infty & \text{sonst} \end{cases}$

$\rightarrow \psi_n = \sqrt{\frac{2}{L}}\sin(k_n x)$ mit $k_n = \frac{n\cdot\pi}{L}$; $E_n = \frac{n^2\pi^2}{L^2}\cdot\frac{\hbar^2}{2m} = \hbar\omega n^2$

$\rightarrow \psi_n(t) \rightsquigarrow \sqrt{\frac{2}{L}}\sin(k_n x)\cdot e^{-i\omega n^2 t}$

$\phi(x, t=0) = \phi(x) = \frac{1}{\sqrt{2}}(\psi_1 + i\,\psi_2)$

$\mapsto \frac{1}{\sqrt{L}}\left(\sin(k_1 x) + i\sin(k_2 x)\right)$

$\rightarrow \phi(x,t) = \frac{1}{\sqrt{2}}\left(\psi_1\cdot e^{-i\omega t} + i\,\psi_2 e^{-i4\omega t}\right)$

$\rightarrow \langle\hat{o}\rangle = \frac{1}{2}\int\left(\psi_1^* e^{i\omega t} - i\,\psi_2^* e^{4i\omega t}\right)\hat{o}\left(\psi_1 e^{-i\omega t} + i\,\psi_2 e^{-i4\omega t}\right)dx$

$= \frac{1}{2}\cdot\int\left[(\psi_1^*\hat{o}\,\psi_1) + (\psi_2^*\hat{o}\,\psi_2) - i\,e^{3i\omega t}\,\psi_2^*\hat{o}\,\psi_1 + i\,e^{-3i\omega t}\,\psi_1^*\hat{o}\,\psi_2\right]dx$

$\int\psi_1^* x\,\psi_1\,dx = \int\psi_2^* x\,\psi_2\,dx = \frac{L}{2}$ weil:

$|\psi_1|^2$ und $|\psi_2|^2$ sind beides gerade Funktionen um $x=\frac{L}{2}$ herum

$\Rightarrow \frac{1}{2}\cdot\left(\frac{L}{2}+\frac{L}{2}\right) + \frac{1}{2}\,i\left(e^{-3i\omega t} - e^{+3i\omega t}\right)\cdot\int\underbrace{\psi_2\,\hat{x}\,\psi_1}\,dx$

weil $\psi_2^* = \psi_2$ und $\psi_1^* = \psi_1$.

$$\frac{L}{2} + \left[\frac{1}{\cancel{2}}\int_0^L \sin\left(\frac{\pi}{L}x\right)\cdot x\cdot \underbrace{\sin\left(\frac{2\pi}{L}x\right)}\,dx\right]\cdot i\left[\cancel{2}i\sin(3\omega t)\right]\cdot\left(\sqrt{\frac{2}{L}}\right)^2$$

$$= \frac{L}{2} + \sin(3\omega t)\cdot\frac{2}{L}\cdot\int_0^L \sin\left(\frac{\pi}{L}x\right)\cdot x\cdot\left(2\sin\left(\frac{\pi}{L}x\right)\cos\left(\frac{\pi}{L}x\right)\right)dx$$

$$= \frac{L}{2} + \frac{4}{L}\sin(3\omega t)\cdot\int_0^L \underbrace{\sin^2\left(\frac{\pi}{L}x\right)\cos\left(\frac{\pi}{L}x\right)}_{u'}\cdot\underbrace{x}_{v}\,dx \quad\rightarrow$$

$$u = \frac{\sin^3\left(\frac{\pi}{L}x\right)\cdot\frac{L}{\pi}}{3} \qquad v' = 1$$

$$\rightarrow \underbrace{\left(\frac{\sin^3\left(\frac{\pi}{L}x\right)L}{3\pi}\cdot x\right)\Bigg|_0^L}_{} - \underbrace{\int_0^L \frac{\sin^3\left(\frac{\pi}{L}x\right)\cdot\frac{L}{\pi}}{3}\,dx}_{}$$

sin( ) gibt an beiden Grenzen $\varnothing$

$$\cos\left(\frac{\pi}{L}x\right) = u$$

$$\frac{\pi}{L}-\sin\left(\frac{\pi}{L}x\right)dx = du$$

$$\sin\left(\frac{\pi}{L}x\right)dx = -\frac{L}{\pi}\,du$$

$x = 0 \rightsquigarrow u = 1$

$x = L \rightarrow u = -1$

$$\cancel{4}\int_1^{-1}(1-u^2)\,du\cdot\frac{L^2}{3\pi^2} = \frac{L^2}{3\pi^2}\cdot\left(u-\frac{u^3}{3}\right)\Bigg|_1^{-1} = -\frac{L^2}{3\pi^2}\cdot\left(-\frac{2}{3}-\frac{2}{3}\right) =$$

$$+\frac{L^2}{3\pi^2}\,\frac{4}{3}$$

$$\left(\frac{L}{2}\right) + \frac{4}{\cancel{L}}\cdot\sin(3\omega t)\cdot\left(-\frac{L^{\cancel{2}}}{9\pi^2}\cdot 4\right) =$$

$$\frac{L}{2} \mp \frac{16}{9\pi^2}L\cdot\sin(3\omega t) = \langle x\rangle(t)$$

Oszilliert mit Frequenz

$$\frac{3\omega}{2\pi} = \left(\frac{E_2-E_1}{\hbar}\right)\cdot\frac{1}{2\pi} = f$$

die Amplitude $\frac{16}{9\pi^2}L$ ist kleiner als der naive erwartete Wert $\frac{L}{2}$ des klassischen Modells

c) $\langle p \rangle (t) \rightarrow$

wieder mit $\frac{1}{2} \cdot \int [(\psi_1^* \hat{o} \psi_1) + (\psi_2^* \hat{o} \psi_2) + i e^{3i\omega t} \psi_2^* \hat{o} \psi_1 + i e^{-3i\omega t} \psi_1^* \hat{o} \psi_2] dx$

$\hat{o} = \frac{\hbar}{i} \partial_x \rightarrow \frac{1}{2} \left[\int_0^L \frac{\hbar}{i} \cdot \frac{2}{L} \sin(\frac{\pi}{L} x) \cdot \cos(\frac{\pi}{L} x) \frac{\pi}{L} dx \right.$

mit Substitution $\sin(\frac{\pi}{L} x) = u$ fallen Grenzen unmittelbar zusammen $\rightarrow 0$

$+ \int_0^L \frac{\hbar}{i} \frac{2}{L} \sin(\frac{2\pi}{L} x) \cdot \cos(\frac{2\pi}{L}) \frac{\pi}{L} dx$

— " — $\sin(\frac{2\pi x}{L}) = u$ — " —

— " — — " —

$- e^{3i\omega t} \cdot \hbar \int_0^L \cdot \frac{2}{L} \cdot \sin(\frac{2\pi}{L} x) \cdot \cos(\frac{\pi}{L} x) dx \cdot \frac{\pi}{L} \cdot \cancel{2}$

$\left. + e^{-3i\omega t} \hbar \int \frac{2}{L} \sin(\frac{\pi}{L} x) \cdot \cos(\frac{2\pi}{L} x) dx \cdot \frac{2\pi}{L} \right] =$

$\frac{1}{\cancel{2}} \cdot \hbar \cdot \frac{2\pi}{L^2} \cdot \left[e^{-3i\omega t} \cdot \cancel{2} \int_0^L \sin(\frac{\pi}{L} x) (\cos^2(\frac{\pi}{L} x) - \sin^2(\frac{\pi}{L} x)) \cdot dx \right.$

$- e^{+3i\omega t} \cdot \int_0^L 2 \cdot \sin(\frac{\pi}{L} x) \cos(\frac{\pi}{L} x) \cos(\frac{\pi}{L} x) dx$

Substitution ist jeweils: $u = \cos(x \cdot \frac{\pi}{L})$

$du = -\sin(\frac{\pi x}{L}) \frac{\pi}{L} dx$

Grenzen $0; L \rightarrow 1; -1$

$\frac{2\pi\hbar}{L^2} \cdot \left[- e^{-3i\omega t} \cdot \int_1^{-1} \cdot \frac{L}{\pi} \cdot (u^2 - (1-u^2)) du + e^{+3i\omega t} \cdot \int_1^{-1} \frac{L}{\pi} \cdot u^2 du \right.$

$\int_1^{-1} u^2 = \left. \frac{u^3}{3} \right|_1^{-1} = -\frac{1}{3} - \frac{1}{3} = -\frac{2}{3}$

$\Rightarrow \frac{2\hbar}{L} \cdot \left[e^{3i\omega t} \cdot (-\frac{2}{3}) - e^{-3i\omega t} \cdot [\overset{\frac{2}{3}}{2 - \frac{4}{3}}] \right] = \frac{2\hbar}{L} \cdot (-\frac{2}{3}) \cdot 2 \cdot \cos(3\omega t)$

$$\rightarrow \langle p\rangle(t) = -\frac{8\hbar}{3L}\cdot\cos(3\omega t)$$

vgl. $\partial_t\langle x\rangle(t) = -\frac{16}{9\pi^2}L\cdot\cos(3\omega t)\cdot 3\omega$ $\qquad \omega = \frac{\pi^2\hbar}{L^2 2m}$

$$= -\frac{8\hbar}{3L\,m}\cos(3\omega t) \rightarrow \partial_t\langle x\rangle(t) \hat{=} \langle v\rangle(t)$$

$$\& \quad \langle v\rangle(t)\cdot m = \langle p\rangle(t)$$

d) mit $\langle\hat{O}\rangle(t)$ wie vorher $\rightarrow$

$$\frac{1}{2}\int\left[(\psi_1^*\mathcal{H}\psi_1) + ie^{3i\omega t}\cdot(\psi_2^*\mathcal{H}\psi_1) + ie^{-3i\omega t}(\psi_1^*\mathcal{H}\psi_2) + (\psi_2^*\mathcal{H}\psi_2)\right]dx$$

da $\mathcal{H}\psi_n = E_n\psi_n$ und die ψ_n eine orthonormalbasis bilden

$$\rightarrow \int\psi_2^*\mathcal{H}\psi_1 dx = \int E_1\psi_2^*\psi_1 dx = \emptyset \quad \text{usw.} \rightsquigarrow$$

$$\frac{1}{2}\left[\int E_1(\psi_1^*\psi_1)dx + \int E_2(\psi_2^*\psi_2)dx\right] = \underline{\underline{\frac{E_2+E_1}{2} \quad \text{zeitunabhängig}}}$$

$\hat{=}$ Energieerhaltung

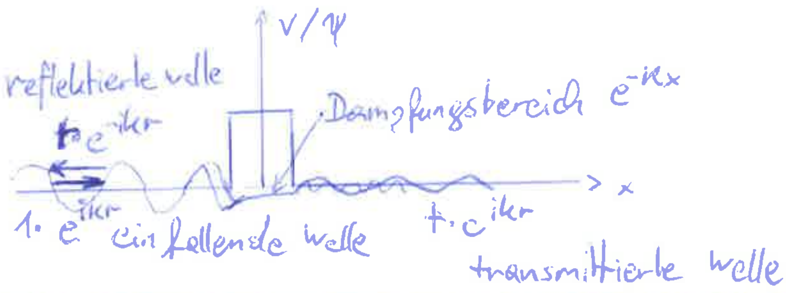
6.)

a) $\mathcal{H} = -\frac{\hbar^2}{2m}\Delta + V(x)$

$$V(x) = \begin{cases} V_0 & \mid -\frac{L}{2} \leq x \leq \frac{L}{2} \\ 0 & \text{sonst} \end{cases} \qquad V_0 \in \mathbb{R}^+$$

$\rightarrow$ Streuung and der Potentialbarriere:

$0 < E < V_0 \rightarrow$



$E = \frac{\hbar^2 k^2}{2m}$ Wellenfkt. im transmissions-Bereich bekannt: $\propto e^{ikr}$

Im Einfallsbereich: $A \cdot e^{ikr} + B \cdot e^{-ikr}$ mit $A, B \in \mathbb{C}$

Innen gilt: $\left(-\frac{\hbar^2}{2m}\Delta + V_0\right)\psi = E\psi$

$$-\frac{\hbar^2}{2m}\Delta\psi = (E - V_0)\psi \qquad (E - V_0) < 0$$

$$\rightarrow \quad \psi = a \cdot e^{\kappa r} + b e^{-\kappa r} \rightarrow \quad a, b \in \mathbb{C}$$

$$-\frac{\hbar^2 \kappa^2}{2m} = (E - V_0)$$

Wellenfunktion und ihre erste Ableitung sind stetig:

$$A \cdot e^{-i\frac{L}{2}k} + B e^{+i\frac{L}{2}k} = a \cdot e^{-\kappa\frac{L}{2}} + b \cdot e^{\kappa\frac{L}{2}} \qquad \text{I}$$

$$a\, e^{\kappa\frac{L}{2}} + b e^{-\kappa\frac{L}{2}} = e^{ik\frac{L}{2}} \qquad \text{II}$$

$$ikA e^{-i\frac{L}{2}k} - ikB e^{i\frac{L}{2}k} = \kappa a e^{-\kappa\frac{L}{2}} - \kappa b e^{\kappa\frac{L}{2}} \qquad \text{III}$$

$$\kappa a e^{\kappa\frac{L}{2}} - \kappa b e^{-\kappa\frac{L}{2}} = ik\, e^{ik\frac{L}{2}} \qquad \text{IV}$$

$\rightarrow \kappa \cdot (\text{II}) - \text{IV} \rightarrow$

$$(\kappa b \cdot 2) e^{-\kappa\frac{L}{2}} = (\kappa - ik) e^{ik\frac{L}{2}}$$

$$b = \frac{(\kappa - ik)\, e^{(ik+\kappa)\frac{L}{2}}}{2\kappa}$$

$\kappa \cdot (\text{II}) + \text{IV} \rightarrow$

$$(\kappa a \cdot 2)\, e^{+\kappa\frac{L}{2}} = (\kappa + ik)\, e^{ik\frac{L}{2}}$$

$$a = \frac{(\kappa + ik)\, e^{(ik-\kappa)\frac{L}{2}}}{2\kappa}$$

$ik\,\text{I} + \text{III} \rightarrow$

$$(ikA \cdot 2) \cdot e^{-i\frac{L}{2}k} = (ik + \kappa) \cdot a\, e^{-\kappa\frac{L}{2}} + (ik - \kappa) b\, e^{\kappa\frac{L}{2}}$$

$$\rightarrow A = \frac{(ik+\kappa) \cdot a\, e^{-\kappa\frac{L}{2}} + (ik - \kappa) b e^{\kappa\frac{L}{2}}}{ik \cdot 2} \cdot e^{+i\frac{L}{2}k}$$

$$A \cdot t = 1 \rightarrow \quad t = \frac{1}{A} \qquad T = |t|^2$$

ik (I) $-$ IV

$$(ik2)B\cdot e^{i\frac{L}{2}k} = (ik-\kappa)a\cdot e^{-\frac{\kappa L}{2}} + (ik+\kappa)b\, e^{\frac{\kappa L}{2}}$$

$$B = \frac{(ik-\kappa)\cdot a\cdot e^{-\frac{\kappa L}{2}} + (ik+\kappa)\cdot b\, e^{+\frac{\kappa L}{2}}}{ik\cdot 2}\cdot e^{-i\frac{L}{2}k}$$

$$B = r\cdot A \quad \rightarrow r = \frac{B}{A} \qquad R = |r|^2$$

$$T+R \overset{!}{=} 1 \quad \rightarrow$$

$$\frac{1}{|A|^2} + \frac{|B|^2}{|A|^2} \overset{!}{=} 1 \quad \rightarrow \quad 1+|B|^2 \overset{!}{=} |A|^2$$

$$B^*\cdot B = \frac{(-ik-\kappa)\cdot a^* e^{-\frac{\kappa L}{2}} + (-ik+\kappa)b^* e^{\frac{\kappa L}{2}}}{-ik\cdot 2}\cancel{e^{+i\frac{L}{2}k}}\cdot \frac{(ik-\kappa)a e^{-\frac{\kappa L}{2}} + (ik+\kappa)\cdot b e^{\frac{\kappa L}{2}}}{ik\cdot 2}\cancel{e^{-i\frac{L}{2}k}}$$

$$= \frac{a^*a\cdot(\kappa^2+k^2)e^{-\kappa L} + a^*b(\kappa+ik)^2\cdot(-1) + b^*a(\kappa-ik)^2(-1) + b^*b(\kappa^2+k^2)\cdot e^{\kappa L}}{4k^2}$$

$$A^*\cdot A = \frac{(-ik+\kappa)\cdot a^* e^{-\kappa\frac{L}{2}} + (-ik-\kappa)b^* e^{\kappa\frac{L}{2}}}{-ik2}\cancel{e^{-i\frac{L}{2}k}}\frac{(ik+\kappa)a\, e^{-\kappa\frac{L}{2}} + (ik-\kappa)b e^{\kappa\frac{L}{2}}}{ik2}\cancel{e^{i\frac{L}{2}k}}$$

$$= \frac{a^*a(\kappa^2+k^2)e^{-\kappa L} + a^*b(\kappa-ik)^2(-1) + b^*a(\kappa+ik)^2(-1) + b^*b(\kappa^2+k^2)e^{\kappa L}}{4k^2}$$

$$\rightarrow A^*A - B^*B = \frac{-a^*b[(\kappa-ik)^2 - (\kappa+ik)^2] - b^*a[(\kappa+ik)^2-(\kappa-ik)^2]}{4k^2} =$$

$$\frac{-a^*b[-2ik\kappa-2ik\kappa] - b^*a[2ik\kappa+2ik\kappa]}{4k^2} =$$

$$\frac{4k}{4k^2}[a^*b\cdot i\cdot\kappa - b^*a i\kappa] = \frac{\kappa}{k}\cdot[a^*bi + (a^*bi)^*] = \frac{\kappa}{k}\cdot 2\mathrm{Re}(a^*bi)$$

$$= \frac{\kappa}{k}\cdot 2\mathrm{Re}\left(i\cdot\frac{(\kappa-ik)e^{(-ik-\kappa)\frac{L}{2}}}{2\kappa}\cdot\frac{(\kappa-ik)\,e^{(ik+\kappa)\frac{L}{2}}}{2\kappa}\right)$$

$$\frac{\kappa}{k}\cdot 2\,\mathrm{Re}\left(i\frac{(\kappa^2-2ik\kappa-k^2)}{4\kappa^2}\right) = \frac{\kappa}{k}\cdot 2\cdot\frac{2k\kappa}{\kappa^2} = 1$$

b) $T = \frac{1}{|A|^2}$

$$|A|^2 = \frac{a^*a(\kappa^2+k^2)e^{-\kappa L} - a^*b(\kappa-ik)^2 - b^*a(\kappa+ik)^2 + b^*b(\kappa^2+k^2)e^{\kappa L}}{4k^2}$$

$$a^*a = \frac{(\kappa-ik)e^{(-ik-\kappa)\frac{L}{2}}}{2\kappa}\;\frac{(\kappa+ik)e^{(ik-\kappa)\frac{L}{2}}}{2\kappa} = \frac{(\kappa^2+k^2)\cdot e^{-\kappa L}}{4\kappa^2}$$

$$b^*b = \frac{(\kappa+ik)e^{(-ik+\kappa)\frac{L}{2}}}{2\kappa}\;\frac{(\kappa-ik)e^{(ik+\kappa)\frac{L}{2}}}{2\kappa} = \frac{(\kappa^2+k^2)e^{\kappa L}}{4\kappa^2}$$

$$a^*b = \frac{(\kappa-ik)e^{(-ik-\kappa)\frac{L}{2}}}{2\kappa}\;\frac{(\kappa-ik)e^{(ik+\kappa)\frac{L}{2}}}{2\kappa} = \frac{(\kappa-ik)^2}{4\kappa^2} = (b^*a)^*$$

$$= \frac{(\kappa^2+k^2)^2e^{-2\kappa L} - (\kappa-ik)^4 - (\kappa+ik)^4 + (\kappa^2+k^2)^2e^{2\kappa L}}{16\kappa^2k^2}$$

$$(\kappa-ik)^4 = (\kappa^2-2ik\kappa-k^2)^2 = (\kappa^2-k^2)^2 - (\kappa^2-k^2)\cdot 4ik\kappa - 4k^2\kappa^2 = ((\kappa+ik)^4)^*$$

$$\frac{(\kappa^2+k^2)^2e^{-2\kappa L} - 2(\kappa^2-k^2)^2 + 8k^2\kappa^2 + (\kappa^2+k^2)^2e^{2\kappa L}}{16\kappa^2k^2}$$

$$-2(\kappa^2-k^2)^2 + 8k^2\kappa^2 = -2(\kappa^2+k^2)^2$$

$$\rightarrow (\kappa^2+k^2)^2(e^{-2\kappa L} - 2 + e^{2\kappa L}) = (\kappa^2+k^2)^2\, 4\sinh^2(\kappa L)$$

$\rightarrow$ Kehrwert:

$$\frac{16\kappa^2k^2}{(\kappa^2+k^2)^2 4\sinh^2(\kappa L) + 16k^2\kappa^2} = \frac{4\kappa^2k^2}{(\kappa^4+2\kappa^2k^2+k^4)\sinh^2(\kappa L) + 4k^2\kappa^2}$$

c) $H \rightarrow \left(-\frac{\hbar^2}{2m}\partial_x\partial_x + \alpha\,\delta(x)\right)$

Im Allgemeinen wird sich Ψ schreiben lassen als:

$$\Theta(x)\Psi_R + \Theta(-x)\Psi_L \rightarrow$$

$$\Theta'(x)\Psi_R'(x) + \Theta''(x)\Psi_R(x) = (\Theta'(x)\Psi_R(x))' = [\delta(x)\Psi_R(x)]' = \delta'(x)\cdot\Psi_R(0)$$

$$-\frac{\hbar^2}{2m}\Big[\big(\Theta(x)\Psi_R''(x) + 2\Theta'(x)\Psi_R'(x) + \Theta''(x)\Psi_R(x)\big) + \big(\Theta(-x)\Psi_L''(x) - 2\Theta'(-x)\Psi_L'(x) + \Theta''(-x)\Psi_L(x)\big)\Big]$$

$$+\alpha\,\delta(x)[\Theta(x)\Psi_R(x) + \Theta(-x)\Psi_L(x)] = E(\Theta(x)\Psi_R(x) + \Theta(-x)\Psi_L(x))$$

an $x \neq 0$ ~>

$$-\frac{\hbar^2}{2m}\Psi_R''(x) = E\Psi_R(x) \qquad -\frac{\hbar^2}{2m}\Psi_L''(x) = E\Psi_L(x) \quad *$$

Wieder besteht die linke Wellenfunktion aus einfallendem und reflektiertem Anteil, Der rechte Anteil nur aus dem transmittierten Anteil

$$\rightarrow \Psi_R = \cancel{A}\cdot \cancel{e}\, e^{ikr} \quad \text{mit} \quad \frac{\hbar^2 k^2}{2m} = E$$

$$\Psi_L = \frac{1}{t}e^{ikr} + \frac{r}{t}e^{-ikr}$$

an $x = 0$ müssen die nach Ordnungen von Divergenz gleichen Terme die Gleichung seperat erfüllen [endliche; δ, δ' Terme]

$$\rightarrow -\frac{\hbar^2}{2m}\left[\Theta(0)\Psi_R''(0) + \cancel{2}\,\delta(0)\Psi_R'(0) + \delta'(0)\Psi_R(0) + \Theta(0)\Psi_L''(0) - \cancel{2}\,\delta(0)\Psi_L'(0) - \delta'(0)\Psi_L(0)\right]$$

$$+ \alpha\delta(0)\left[\Theta(0)\Psi_R(0) + \Theta(0)\Psi_L(0)\right] = E(\Theta(0)\Psi_R(0) + \Theta(0)\Psi_L(0))$$

Dabei wurde verwendet $\delta'(-x) = -\delta'(x)$ da δ' eine ungerade Funktion ist.
Mögliche Definition für $\Theta(0) \rightsquigarrow \frac{1}{2}$ (aus Konsistenz)

$$-\frac{\hbar^2}{2m}\left[\frac{1}{2}\Psi_R''(0) + \frac{1}{2}\Psi_L''(0)\right] = E\left[\frac{1}{2}\Psi_R(0) + \frac{1}{2}\Psi_L(0)\right] \quad \text{ist erfüllt wegen } *$$

$$-\frac{\hbar^2}{2m}\left[\cancel{1}\,\delta(0)\left[\Psi_R'(0) - \Psi_L'(0)\right]\right] + \alpha\delta(0)\frac{1}{2}\left[\Psi_L(0) + \Psi_R(0)\right] = 0$$

$$-\frac{\hbar^2}{2m}\delta'(0)\cdot\left[\Psi_R(0) - \Psi_L(0)\right] = 0 \rightarrow \Psi_R(0) = \Psi_L(0) := \Psi(0) \rightarrow \Psi \text{ ist stetig}$$

$$\rightarrow -\frac{\hbar^2}{2m}\cdot\cancel{1}\cdot\left[\Psi_R'(0) - \Psi_L'(0)\right] = -\alpha\Psi(0) \rightarrow \checkmark$$

d) $\frac{1}{t} + \frac{r}{t} = 1$ aus Stetigkeit $\rightarrow$ $\frac{1+r}{t} = 1$ $\quad 1+r = t \rightarrow r = t-1$

$$-\frac{\hbar^2}{2m}\cdot\left[ik - \left(ik\frac{1}{t} - ik\frac{r}{t}\right)\right] = -\alpha \quad \text{aus Sprungbedingung in } \Psi'$$

$$\frac{ik\hbar^2}{2m}\left[1 + \frac{r}{t} - \frac{1}{t}\right] = \cancel{\alpha}$$

$$\left(\frac{t-1+r}{t}\right) = \frac{2m}{ik\hbar^2}\alpha$$

$$\frac{2r}{t} = \frac{2m}{ik\hbar^2}\alpha \rightarrow \cancel{2}\,\frac{t-1}{t} = \frac{m}{ik\hbar^2}\alpha \rightarrow \quad -\frac{1}{t} = \frac{m\alpha}{ik\hbar^2} - 1$$

$$\frac{1}{t} = 1 - \frac{m\alpha}{ik\hbar^2}$$

$t = \frac{ik\hbar^2}{ik\hbar^2 - m\alpha} = 1 + \frac{m\alpha}{ik\hbar^2 - m\alpha} \rightarrow r = \frac{m\alpha}{ik\hbar^2 - m\alpha}$

$\rightarrow T = |t|^2 = \frac{k^2\hbar^4}{k^2\hbar^4 + m^2\alpha^2} \qquad R = |r|^2 = \frac{m^2\alpha^2}{k^2\hbar^4 + m^2\alpha^2}$

Wenn $E \rightarrow 0$ geht $\rightarrow k^2 \rightarrow 0 \rightsquigarrow T \rightarrow 0$; $R \rightarrow 1$

$E \rightsquigarrow \infty \rightarrow k^2 \rightarrow \infty \rightsquigarrow T \rightarrow 1$; $R \rightarrow 0$

e) $L \rightarrow 0$ $\quad L \cdot V_0$ = Wert des Integrals über $V(x)$

$\rightarrow \alpha = L \cdot V_0$

$K = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)} \rightsquigarrow \sqrt{\frac{2m}{\hbar^2} V_0} \Rightarrow \sqrt{\frac{2m\alpha}{\hbar^2 L}}$

$\rightarrow \sinh^2(KL) = \sinh^2\left(\sqrt{\frac{2m\alpha}{\hbar^2}}\sqrt{L}\right) \rightsquigarrow \frac{2m\alpha}{\hbar^2} \cdot L$

$\frac{4m^2\alpha^2}{\hbar^4} \cdot \frac{1}{K^2}$

$L = \frac{2m\alpha}{\hbar^2 \cdot K^2}$

Transmissionskoeffizient:

$\frac{16K^2k^2}{(K^2+k^2)^2 \cdot 4\sinh^2(KL) + 16K^2k^2}$

$K^2 = \frac{2m\alpha}{\hbar^2 L} \rightsquigarrow \infty$

$\Rightarrow K^4 \cdot \frac{1}{K^2}$ Ordnung $\rightsquigarrow$ dominiert Nenner

K^2 Ordnung

$\rightarrow \frac{16 \cdot \frac{2m\alpha}{\hbar^2 L} \cdot k^2}{4 \cdot \left(K^4 \cdot \frac{4m^2\alpha^2}{\hbar^4} \frac{1}{\cancel{K^2}}\right) + 16K^2k^2} = \frac{\frac{2m\alpha}{\hbar^2 L} \cdot k^2}{\frac{2m\alpha}{\hbar^2 L} \cdot \frac{m^2\alpha^2}{\hbar^4} + \frac{2m\alpha}{\hbar^2 L} k^2} = \frac{k^2}{\frac{m^2\alpha^2}{\hbar^4} + k^2}$

$\rightarrow$ Im Limes gleich!