Test A Musterlösung bzw Test B 3.)

1) Verständnisfragen

a) $\Theta = i\,|\uparrow\rangle\langle\downarrow| - i\,|\downarrow\rangle\langle\uparrow|$ $\Rightarrow$

b $\big(i\,|\downarrow\rangle\langle\uparrow| - i\,|\uparrow\rangle\langle\downarrow|\big)$

EW	EF
$+1$ (-1)	$\frac{1}{\sqrt{2}}\big(|\uparrow\rangle - i|\downarrow\rangle\big)$
-1 $(+1)$	$\frac{1}{\sqrt{2}}\big(|\uparrow\rangle + i|\downarrow\rangle\big)$

Wahrsch. $+1$ (-1) zu messen:

$$\left|\left(\frac{1}{\sqrt{2}}\langle\uparrow| + i\langle\downarrow|\right)\cdot \frac{1}{\sqrt{10}}\underbrace{\big(3|\uparrow\rangle + i|\downarrow\rangle\big)}_{|\psi\rangle}\right|^2 = \left|\frac{3}{\sqrt{20}} + \frac{i^2}{\sqrt{20}}\right|^2$$

$\big(|\uparrow\rangle - 3i|\downarrow\rangle\big)$ $\quad \left|\left(\frac{1}{\sqrt{20}} + \frac{3}{\sqrt{20}}\right)\right|^2$

$$= \frac{4}{20} = \frac{1}{5} \qquad \frac{16}{20} = \frac{4}{5}$$

$\Rightarrow$ Wahrsch -1 $(+1)$ zu messen: $1 - \frac{1}{5} = \frac{4}{5}$ $\quad \frac{1}{5}$

b) EF zu EW $+1$ von A (Θ): $|\psi\rangle = \dfrac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$

(wird mit Wahrsch. 100% EW $+1$ liefern $\Rightarrow \langle A\rangle = 1$) $\langle\Theta\rangle = 1$

c) linear immer $\forall \lambda \in \mathbb{C}$ da A linear $\Rightarrow A^2$, $e^{\lambda A}$ linear

hermitesch

$\Theta = e^{\lambda A} \overset{!}{=} \Theta^\dagger = e^{\lambda^* A}$

$\Leftrightarrow \lambda = \lambda^* \Leftrightarrow \lambda \in \mathbb{R}$

$\Theta' = (\lambda A)^2 \overset{!}{=} (\Theta')^\dagger = (\lambda^* A)^2$ $\quad i \quad -i$

$\Leftrightarrow \lambda^2 = \pm\lambda^{*2} \Rightarrow \lambda^2 \in \mathbb{R}$

Sei $\lambda = |\lambda|\, e^{i\varphi}$ $\quad |\lambda|, \varphi \in \mathbb{R}$

$\Rightarrow \lambda^2 = e^{2i\varphi} \overset{!}{=} \pm\lambda^{*2} = e^{-2i\varphi}$

$\Rightarrow 4\varphi = 2\pi n \quad n \in \mathbb{Z}$

$\varphi = \frac{\pi n}{2} \quad n \in \mathbb{Z}$

$|\lambda| \in \mathbb{R}$

Test B:

$A \to \Theta$

unitär

$\Theta\Theta^\dagger = e^{\lambda A} e^{\lambda^* A} = e^{(\lambda+\lambda^*)A}$

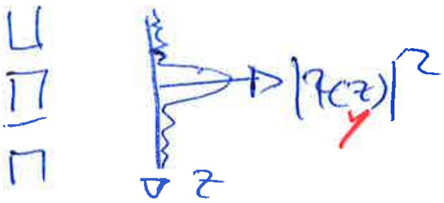
$\Rightarrow \lambda + \lambda^* = 0 \quad \lambda = ic \quad c \in \mathbb{R}$

$\Theta'(\Theta')^\dagger = (\lambda A^\dagger)^2 (\lambda^* A)^2$ $\quad i \quad i$

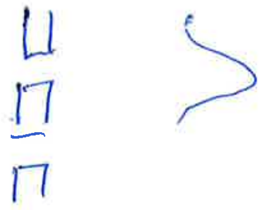
für beliebige A nicht unitär

(A kann EW 0 haben)

d) ohne Messung



Messung 0 d.h. oben



Messung 1

mit Messapparat

e) $E\,\psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\,\psi(x) \qquad x > 0$

$$\frac{\partial^2}{\partial x^2}\psi(x) = \frac{\partial}{\partial x}\left[A\,n x^{n-1} e^{-\gamma x} + A x^n (-\gamma) e^{-\gamma x}\right]$$

$$= A\left[n(n-1)\,x^{n-2} e^{-\gamma x} + 2An x^{n-1}(-\gamma) e^{-\gamma x} + A x^n (-\gamma)^2 e^{-\gamma x}\right]$$

$$\Rightarrow V(x)\cdot A x^n e^{-\gamma x} = E A x^n e^{-\gamma x} + \frac{\hbar^2}{2m} A x^n e^{-\gamma x} \times\left[\frac{n(n-1)}{x^2} + \frac{2n(-\gamma)}{x} + \gamma^2\right]$$

$: A x^n e^{-\gamma x}$

$$V(x) - E = \frac{\hbar^2}{2m}\left[\underbrace{\frac{n(n-1)}{x^2}}_{\xrightarrow{x\to\infty} 0} + \underbrace{\frac{2n(-\gamma)}{x}}_{\xrightarrow{x\to\infty} 0} + \gamma^2\right]$$

$$V(x) \xrightarrow{x\to\infty} 0 \qquad \Rightarrow E = -\gamma^2\frac{\hbar^2}{2m}$$

$$V(x) = \frac{\hbar^2}{2m}\left[\frac{n(n-1)}{x^2} + \frac{2n(-\gamma)}{x}\right]$$

2.) Kinetische Energie [B: Virialtheorem]

a)
$$[x,H] = [x, p^2/2m] = \frac{1}{2m}[x,p^2] = \frac{1}{2m}(xp^2 - pxp + pxp - p^2x)$$
$$= \frac{1}{2m}\left([x,p]p + p[x,p]\right) = \frac{i\hbar p}{m}$$

$\varepsilon_n - \varepsilon_{n'}$ in Test B

$$\langle\phi_n|p|\phi_{n'}\rangle = -\frac{im}{\hbar}\langle\phi_n|[x,H]|\phi_{n'}\rangle = +\frac{im}{\hbar}(E_n - E_{n'})\langle\phi_n|x|\phi_{n'}\rangle$$

α

b)
$$\frac{\hbar^2}{m^2}\langle\phi_n|p^2|\phi_n\rangle = \frac{\hbar^2}{m^2}\sum_{n'}\langle\phi_n|p|\phi_{n'}\rangle\langle\phi_{n'}|p|\phi_n\rangle$$

[bzw. γ in Test B]

$$= \frac{\hbar^2}{m^2}\cdot\frac{m^2}{\hbar^2}\sum_{n'}(E_n - E_{n'})^2\langle\phi_n|x|\phi_{n'}\rangle\langle\phi_{n'}|x|\phi_n\rangle$$
$$= \sum_{n'}(E_n - E_{n'})^2\,|\langle\phi_n|x|\phi_{n'}\rangle|^2$$

c)
$$\langle T\rangle_n = \langle\phi_n|\frac{p^2}{2m}|\phi_n\rangle \quad \text{mit} \quad H|\phi_n\rangle = \overbrace{\hbar\omega\left(n+\tfrac{1}{2}\right)}^{E_n}|\phi_n\rangle$$

$$\langle T\rangle_n = \frac{m}{2\hbar^2}\frac{\hbar^2}{m^2}\langle\phi_n|p^2|\phi_n\rangle = \frac{m}{2\hbar^2}\sum_{n'}(E_n - E_{n'})^2|\langle\phi_n|x|\phi_{n'}\rangle|^2$$
$$= \frac{m}{2\hbar^2}\hbar^2\omega^2\sum_{n'}\left(n+\cancel{\tfrac{1}{2}}-n'-\cancel{\tfrac{1}{2}}\right)^2|\langle\phi_n|x|\phi_{n'}\rangle|^2$$
$$= \frac{m\omega^2}{2}\sum_{n'}(n-n')^2\frac{\hbar}{2m\omega}|\langle\phi_n|a+a^+|\phi_{n'}\rangle|^2$$
$$= \frac{\hbar\omega}{4}\sum_{n'}(n-n')^2\left|\underbrace{\langle\phi_n|a|\phi_{n+1}\rangle}_{\sqrt{n+1}\,\delta_{n',n+1}} + \underbrace{\langle n|a^+|n-1\rangle}_{\sqrt{n}\,\delta_{n',n-1}}\right|^2$$
$$= \frac{\hbar\omega}{4}(n+1+n) = \frac{\hbar\omega}{4}(2n+1) = \frac{\hbar\omega}{2}\left(n+\frac{1}{2}\right)$$

Virialtheorem: $\langle T\rangle_n = \frac{1}{2}\langle H\rangle_n$! $= \varepsilon_n/2$!

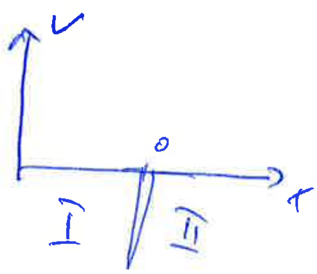
Test 1, Musterlösung Aufg 3 (A)

1 (B)

$V(x) = -\cancel{\gamma}\ \delta\left(\frac{2m}{\hbar^2}x\right)$, $\cancel{\gamma} > 0$

$+b$ $\quad\quad b < 0$

genereller Ansatz:



$$\mathrm{I}: \quad \psi_{\mathrm{I}}(x) = A e^{ikx} + B e^{-ikx}$$

$$\mathrm{II}: \quad \psi_{\mathrm{II}}(x) = C e^{ikx} + D e^{-ikx}$$

freies Problem in I, II $\rightarrow$ $k = \sqrt{\frac{2mE}{\hbar^2}}$

Anschlußbedingungen:

$$\psi_{\mathrm{I}}(0) = \psi_{\mathrm{II}}(0) \qquad (1)$$

$$\psi_{\mathrm{I}}'(0) - \psi_{\mathrm{II}}'(0) = \underset{-b}{+\cancel{\gamma}}\ \psi(0) \qquad (2)$$

$$\rightarrow \quad A + B = C + D \qquad (1')$$

$$ik\left[A + D - (B + C)\right] = (A+B)\ \underset{(-b)}{\cancel{\gamma}} \qquad (2')$$

a) von links einlaufende ebene Welle

$\rightarrow$ $A = 1$, $B = r$

$C = t$, $D = 0$

$(1')$ $\rightarrow$ $1 + r = t$

$(2')$ $\rightarrow$ $ik\left[1 - \underbrace{(r + t)}_{2r+1}\right] = (1+r)\ \underset{(-b)}{\cancel{\gamma}}$

$\rightarrow$ $r = -\frac{\overset{(-b)}{\cancel{\gamma}}}{2ik + \underset{-b}{\cancel{\gamma}}}$, $t = 1 - \frac{\overset{(-b)}{\cancel{\gamma}}}{2ik + \underset{-b}{\cancel{\gamma}}} = \frac{2ik}{2ik + \cancel{\gamma}}$ $\quad \frac{2ik}{2ik - b}$

1

a) $\boxed{\begin{aligned} \psi_I(x) &= e^{ikx} - \frac{\gamma}{2ik+\gamma} e^{-ikx} \;\Big\|\; e^{ikx} + \frac{b}{2ik-b} e^{-ikx} \\ \psi_{II}(x) &= \frac{2ik}{2ik+\gamma} e^{+ikx} \;\Big\|\; \frac{2ik}{2ik-b} e^{ikx} \qquad k = \sqrt{\frac{2mE}{\hbar^2}} \end{aligned}}$

b) $\boxed{\begin{aligned} R &= |r|^2 = \frac{\gamma^2}{\gamma^2+4k^2} \;\Big\|\; \frac{b^2}{b^2+4k^2} \\ T &= |t|^2 = \frac{4k^2}{\gamma^2+4k^2} \;\Big\|\; \frac{4k^2}{b^2+4k^2} \qquad R + T = 1 \; \checkmark \end{aligned}}$

c) gebundene Zustände

Ansatz
$$\psi_I(x) = A e^{\kappa x} \qquad x < 0$$
$$\psi_{II}(x) = B e^{-\kappa x} \qquad x > 0 \qquad , \quad \kappa = \sqrt{\frac{-2mE}{\hbar^2}}$$
$$E < 0$$

(1) $\to$ $A = B$ ~~oder Potentialsymmetrie~~

(2) $\to$ $A\kappa + \overset{A}{\overset{\shortparallel}{B}}\kappa = \overset{(-b)}{\gamma} A \iff \kappa = \frac{\overset{(-b)}{\gamma}}{2} \left(= \sqrt{\frac{-2mE}{\hbar^2}} \right)$

$\to$ $\boxed{E = -\frac{\hbar^2}{2m}\frac{\gamma^2}{4} \to 1 \text{ gebundener Zustand.}}$ $E = -\frac{\hbar^2}{2m} b^2/4$

Normierung
$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 2\int_0^{\infty} |\psi_{II}(x)|^2 dx = 2A^2\int_0^{\infty} e^{-2\kappa x} dx$$
$$= -\frac{A^2}{\kappa} e^{-2\kappa x}\Big|_0^{\infty} = \frac{A^2}{\kappa} \overset{!}{=} 1 \iff A = \sqrt{\kappa}$$
$$= \sqrt{\frac{\gamma}{2}}$$
$$A = \sqrt{\frac{-b}{2}}$$

Zeitentwicklung
$$\boxed{\psi(x,t) = e^{-\frac{i}{\hbar}Ht}\psi(x) = \sqrt{\frac{\gamma}{2}}\, e^{\frac{i\hbar}{8m}\gamma^2 t}\, e^{-\frac{\gamma}{2}|x|}}$$
$$\sqrt{\frac{-b}{2}}\, e^{\frac{i\hbar}{8m} b^2 t}\, e^{+\frac{b|x|}{2}}$$

Musterlösung Aufgabe 4 TEST A

$$\Psi(x,t=0) = \frac{2}{3}\psi_0(x) + \frac{2}{3}\psi_1(x) - \frac{i}{3}\psi_3(x)$$

Dirac notation

$$|\Psi(t=0)\rangle = \frac{2}{3}|0\rangle + \frac{2}{3}|1\rangle - \frac{i}{3}|3\rangle$$

a)

$$H|n\rangle = \hbar\omega\left(n+\frac{1}{2}\right)|n\rangle \quad \text{(Harmonischer Oszillator)}$$

$$|\Psi(t)\rangle = \frac{2}{3}e^{-i\frac{1}{2}\omega t}|0\rangle + \frac{2}{3}e^{-i\frac{3}{2}\omega t}|1\rangle - \frac{i}{3}e^{-i\frac{7}{2}\omega t}|3\rangle$$

b) $t=0$: Messwerte: $E_0 = \frac{1}{2}\hbar\omega$, $\quad p_0 = \left|\frac{2}{3}\right|^2 = \frac{4}{9}$

$E_1 = \frac{3}{2}\hbar\omega \quad p_1 = \left|\frac{2}{3}\right|^2 = \frac{4}{9}$

$E_3 = \frac{7}{2}\hbar\omega \quad p_3 = \left|\frac{-i}{3}\right|^2 = \frac{1}{9}$

$t>0$: Messwerte auch E_0, E_1, E_3

$$p_0 = \left|\frac{2}{3}e^{-i\frac{1}{2}\omega t}\right|^2 = \frac{4}{9} \; ; \; p_1 = \left|\frac{2}{3}e^{-i\frac{3}{2}i\omega t}\right| = \frac{4}{9}$$

$$p_3 = \left|-\frac{i}{3}e^{-i\frac{7}{2}\omega t}\right|^2 = \frac{1}{9}$$

Zeitunabhängig!

$$\langle E\rangle = \sum_i E_i p_i = \frac{4}{9}\cdot\frac{1}{2}\hbar\omega + \frac{4}{9}\cdot\frac{3}{2}\hbar\omega + \frac{1}{9}\cdot\frac{7}{2}\hbar\omega = \frac{23}{18}\hbar\omega$$

Zeitunabhängig!

c) $\langle x \rangle (t) = ?$

$$x = \frac{x_0}{\sqrt{2}}(a + a^+) \quad \text{mit} \quad x_0 = \sqrt{\frac{\hbar}{m\omega}}$$

$$\langle x \rangle = \langle \Psi(t) | \underbrace{\frac{x_0}{\sqrt{2}}(a + a^+)}_{} | \Psi(t) \rangle =$$

nur elemente mit $\Delta n = \pm 1$ werden nicht 0

$$= \frac{x_0}{\sqrt{2}} \Big(\frac{2}{3} e^{i\frac{1}{2}\omega t} \langle 0 | a \, \frac{2}{3} e^{-i\frac{3}{2}\omega t} | 1 \rangle +$$

$$+ \frac{2}{3} e^{i\frac{3}{2}\omega t} \langle 1 | a^+ \, \frac{2}{3} e^{i\frac{1}{2}\omega t} | 0 \rangle \Big) =$$

$$= \frac{4x_0}{9\sqrt{2}} \Big(e^{-i\omega t} \langle 0 | \underbrace{a | 1 \rangle}_{|0\rangle} + e^{i\omega t} \langle 1 | \underbrace{a^+ | 0 \rangle}_{|1\rangle} \Big) =$$

$$= \frac{4x_0}{9\sqrt{2}} \left(e^{-i\omega t} + e^{i\omega t} \right) = \frac{8x_0}{9\sqrt{2}} \cos \omega t$$

$$\langle p^2 \rangle (t) = \langle \Psi(t) | p^2 | \Psi(t) \rangle = ?$$

$$p = \frac{\hbar}{i x_0 \sqrt{2}} (a - a^+)$$

$$p^2 = \frac{-\hbar^2}{2x_0^2} (a^2 - a^+ a - \underbrace{a a^+}_{a^+ a + 1} + a^{+2}) = \frac{-\hbar^2}{2x_0^2} \left(a^2 - (2\hat{n} + 1) + a^{+2} \right)$$

mit $\hat{n} = a^+ a$

$\hat{n} |n\rangle = n |n\rangle$

Es gibt Beiträge mit $\Delta n = \pm 2$ und mit $\Delta n = 0$.

Mit $\Delta n = 0$:

$$\langle \Psi(t) | (2\hat{n} + 1) | \Psi(t) \rangle = \frac{2}{3} e^{+i\frac{1}{2}\omega t} \langle 0 | \frac{2}{3} e^{-i\frac{1}{2}\omega t} (2\hat{n} + 1) | 0 \rangle +$$

$$+ \frac{2}{3} e^{+i\frac{3}{2}\omega t} \langle 1 | \frac{2}{3} e^{-i\frac{3}{2}\omega t} (2\hat{n} + 1) | 1 \rangle + \frac{i}{3} e^{i\frac{7}{2}\omega t} \langle 3 | \left(\frac{-i}{3}\right) e^{-i\frac{7}{2}\omega t} (2\hat{n} + 1) | 3 \rangle$$

$$\langle\Psi(t)|(2\hat{n}+1)|\Psi(t)\rangle = \frac{4}{9}\langle 0|(2\hat{n}+1)|0\rangle + \frac{4}{9}\langle 1|(2\hat{n}+1)|1\rangle +$$

$$+\frac{1}{9}\langle 3|(2\hat{n}+1)|3\rangle = \frac{4}{9} + 3\cdot\frac{4}{9} + \frac{1}{9}\cdot 7 = \frac{23}{9}$$

Mit $\Delta n = \pm 2$

$$\langle\Psi(t)|(a^2 + a^{+2})|\Psi(t)\rangle = \frac{2}{3}e^{i\frac{3}{2}\omega t}\langle 1|\left(-\frac{i}{3}\right)e^{-i\frac{7}{2}\omega t}\underbrace{a^2|3\rangle}_{\underbrace{a\sqrt{3}|2\rangle}_{\sqrt{2}\sqrt{3}|1\rangle}} +$$

$$+\frac{i}{3}e^{i\frac{7}{2}\omega t}\langle 3|\frac{2}{3}e^{-i\frac{3}{2}\omega t}\underbrace{a^{+2}|1\rangle}_{a^+\sqrt{2}|2\rangle = \sqrt{3}\sqrt{2}|3\rangle} =$$

$$= -\frac{2i}{9}e^{-2i\omega t}\sqrt{6} + \frac{2i}{9}e^{2i\omega t}\sqrt{6} = \frac{2i\sqrt{6}}{9}\cdot 2i\sin(2\omega t) =$$

$$= -\frac{4\sqrt{6}}{9}\sin(2\omega t)$$

$$\langle p^2(t)\rangle = -\frac{\hbar^2}{2x_0^2}\left(-\frac{23}{9} - \frac{4\sqrt{6}}{9}\sin(2\omega t)\right) = \hbar m\omega\left(\frac{23}{18} + \frac{2\sqrt{6}}{9}\sin(2\omega t)\right)$$

d)

$$\rho(x,t) = |\Psi(x,t)|^2\,dx = |\langle x|\Psi(t)\rangle|^2\,dx$$

$$p(0,t) = |\Psi(0,t)|^2\,dx = \frac{4}{9}|\Psi_0(0,t)|^2\,dx = \frac{4}{9}\frac{1}{\sqrt{\pi}x_0}\,dx$$

Nur Ψ_0 Beitrag bleibt, weil
$\Psi_1(x=0) = \Psi_3(x=0) = 0$ (antisymmetrisch)

Zeitunabhängig!

Musterlösung Aufgabe 4 Test B (1)

$$\Psi(x, t=0) = \frac{1}{2}\Psi_0(x) + \frac{i}{2}\Psi_1(x) + \frac{\sqrt{2}}{2}\Psi_3(x)$$

a) $$|\Psi(t)\rangle = \frac{1}{2} e^{-i\frac{1}{2}\omega t}|0\rangle + \frac{i}{2} e^{-i\frac{3}{2}\omega t}|1\rangle + \frac{\sqrt{2}}{2} e^{-i\frac{7}{2}\omega t}|3\rangle$$

b) Wie in Test A, für $t=0$ und $t>0$ die Messwerte und Wahrscheinlichkeiten sind gleich (Zeitunabhängig)

$$\left.\begin{aligned} E_0 &= \frac{1}{2}\hbar\omega \ ; & P_0 &= \left|\frac{1}{2}\right|^2 = \frac{1}{4} \\ E_1 &= \frac{3}{2}\hbar\omega \ ; & P_1 &= \left|\frac{i}{2}\right|^2 = \frac{1}{4} \\ E_3 &= \frac{7}{2}\hbar\omega \ ; & P_3 &= \left|\frac{\sqrt{2}}{4}\right|^2 = \frac{1}{2} \end{aligned}\right\} \langle E\rangle = \frac{1}{4}\cdot\frac{1}{2}\hbar\omega + \frac{1}{4}\cdot\frac{3}{2}\hbar\omega + \frac{1}{2}\cdot\frac{7}{2}\hbar\omega = \frac{9}{4}\hbar\omega$$

c) Wie in Test A mit anderen Vorfaktoren

$$\langle\Psi(t)|\frac{x_0}{\sqrt{2}}(a+a^+)|\Psi(t)\rangle = \frac{x_0}{\sqrt{2}}\left[\frac{1}{2}e^{\frac{1}{2}i\omega t}\langle 0|a\,\frac{i}{2}e^{-i\frac{3}{2}\omega t}|1\rangle + \right.$$

$$\left. + \frac{i}{2}e^{i\frac{3}{2}\omega t}\langle 1|\frac{1}{2}e^{-i\frac{1}{2}\omega t}a^+|0\rangle\right] = \frac{i x_0}{4\sqrt{2}}\left[e^{-i\omega t}\langle 0|\underbrace{a|1\rangle}_{|0\rangle} - e^{i\omega t}\langle 1|\underbrace{a^+|0\rangle}_{|1\rangle}\right] =$$

$$= \frac{x_0}{2\sqrt{2}}\sin(\omega t)$$

$$p^2 = \frac{-\hbar^2}{2x_0^2}\left(a^2 - (2\hat{n}+1) + a^{+2}\right)$$

für $\Delta n = 0$

die Zeitfaktoren kürzen sich wie im Test A

$$\langle\Psi(t)|(2\hat{n}+1)|\Psi(t)\rangle = \frac{1}{4}\langle 0|(2\hat{n}+1)|0\rangle + \frac{1}{4}\langle 1|(2\hat{n}+1)|1\rangle +$$

$$+ \frac{1}{2}\langle 3|(2\hat{n}+1)|3\rangle = \frac{1}{4} + \frac{1}{4}\cdot 3 + \frac{1}{2}\cdot 7 = \frac{9}{2}$$

Für $\Delta n = \pm 2$

②

$$\langle \Psi(t) | (a^2 + a^{+2}) | \Psi(t) \rangle = -\frac{i\sqrt{2}}{4} e^{-2i\omega t} \langle 1 | \underbrace{a^2 | 3 \rangle}_{a\sqrt{3}|2\rangle = \sqrt{2}\sqrt{3}|1\rangle} +$$

$$+ \frac{i\sqrt{2}}{4} e^{2i\omega t} \langle 3 | \underbrace{a^{+2} | 1 \rangle}_{a^+\sqrt{2}|2\rangle = \sqrt{3}\sqrt{2}|3\rangle} =$$

$$= \frac{i\sqrt{2}\sqrt{6}}{4}\left(e^{2i\omega t} - e^{-2i\omega t}\right) = -\sqrt{3}\sin(2\omega t)$$

$$\langle \Psi(t) | p^2 | \Psi(t) \rangle = \frac{-\hbar^2}{2x_0^2}\left(-\frac{9}{2} - \sqrt{3}\sin(2\omega t)\right) = \frac{\hbar m\omega}{2}\left(\frac{9}{2} + \sqrt{3}\sin(2\omega t)\right)$$

d) Gleich wie im Test A:

$$p(0,t) = |\Psi(0,t)|^2 dx = \frac{1}{4}|\Psi_0(0,t)|^2 dx = \frac{1}{4}\frac{1}{\sqrt{\pi}\,x_0}\,dx$$

<u>Bemerkung</u> Man könnte für die Berechnung von $\langle n | p^2 | n \rangle$ Virialtheorem aus 2b) benutzen

$$\langle T \rangle_n = \langle n | \frac{p^2}{2m} | n \rangle = \frac{1}{2}\langle H \rangle_n$$

$$\langle n | p^2 | n \rangle = m\langle H \rangle_n = m\hbar\omega\left(n + \frac{1}{2}\right)$$