Daniel Grumiller March 8th 2010

Black Holes II — Exercise sheet 2

(12.1) **Penrose diagram for Robinson–Bertotti**
We derived last semester (see exercise 9.1) the near horizon limit geometry of the extremal Reissner–Nordström black hole, which is known as Robinson–Bertotti geometry. Its line element is given by

$$\mathrm{d}s^2 = -\lambda^2\, \mathrm{d}t^2 + Q^2\, \frac{\mathrm{d}\lambda^2}{\lambda^2} + Q^2\, \mathrm{d}\Omega^2_{S^2}$$

where Q is a constant (the charge) and $\mathrm{d}\Omega^2_{S^2}$ is the line-element of the round S^2. Show that the singularity at $\lambda = 0$ is merely a coordinate singularity. Show further that $\lambda = 0$ is a degenerate Killing horizon with respect to ∂_t. Finally, obtain the maximal analytic extension of the Robinson–Bertotti metric and deduce its Penrose diagram.

(12.2) **Inventing Penrose diagrams**
Draw (2-dimensional) Penrose diagrams for spacetimes with the following properties:

(a) Asymptotically flat, event horizon, no singularity

(b) Asymptotically flat, as many Killing horizons as possible, no Cauchy horizon

(c) Asymptotically flat, no event horizon, singularity

(d) Asymptotically flat, two non-extremal and one extremal Killing horizon

(e) Asymptotically flat, at least one Killing horizon, no singularity, no event horizon

(12.3) **Null completeness**
Suppose there was some black hole similar to the Schwarzschild black hole, i.e., it is asymptotically flat, has an event horizon that is a Killing horizon and behind that horizon there is a singularity, in the sense that time-like geodesics end there with finite affine parameter. Suppose further an important difference to the Schwarzschild case, namely completeness with respect to null geodesics. What would happen if the event horizon of this black hole went right through you *now* ? What would happen if you tried to send a message with your mobile phone to the singularity? What would happen if you threw the mobile phone into the singularity? Could such a black hole exist mathematically? Could such a black hole exist in Nature?

These exercises are due on March 15th 2010.

Hints:

- For the first two questions the coordinate transformation $u = t + Q/\lambda$, $v = t - Q/\lambda$ is helpful. For the final task the coordinate transformation $u = \tan(U/2)$, $v = -\cot(V/2)$ is convenient.
- Follow the algorithm explained during the lectures: start with the asymptotically flat region and "design" an Eddington–Finkelstein patch such that all requirements of the sub-exercise are met. Then, if possible, glue together copies of this Eddington–Finkelstein patch (and/or flipped versions thereof).
- No hints this time.