Daniel Grumiller April 19th 2010

Black Holes II — Exercise sheet 5

(15.1) **The ab-family of solutions**

Consider 2D dilaton gravity models

$$S^{\text{2DG}} = \frac{1}{\kappa}\int \mathrm{d}^2x\sqrt{-g}\,\Big[XR - U(X)(\nabla X)^2 - 2V(X)\Big]$$

with the following 2-parameter family of potentials:

$$U(X) = -\frac{a}{X} \qquad V(X) = -\frac{1}{2}\,X^{a+b}$$

Find all classical solutions of that theory in an Eddington–Finkelstein patch. Discuss what happens in the following three cases

(i) $a = 1 + b$

(ii) $a = 1 - b$

(iii) $b = 0$

What is the corresponding ground state geometry in each of these three special cases?

(15.2) **Jackiw–Teitelboim, Katanaev–Volovich, CGHS et al.**

Consider 2D dilaton gravity (see above) with the following potentials

(i) $U = 0$, $V = -\Lambda X$ (Jackiw–Teitelboim)

(ii) $U = \alpha$, $V = \beta X^2 - \lambda$ (Katanaev–Volovich)

(iii) $U = 0$, $V = -\frac{1}{2}$ (Callan–Giddings–Harvey–Strominger)

(iv) $U = a$, $V = e^{\alpha X}$ (Liouville gravity)

(v) $U = 0$, $V = \frac{1}{2}\,X(c - X^2)$ (Kaluza–Klein reduced Chern–Simons)

Which of these models belong to the ab-family of exercise (15.1)? Which of these models have constant dilaton vacua [see exercise (15.3)]? Which of these models have a Minkowski ground state? How many Killing horizons are there for each model?

(15.3) **Constant dilaton vacua**

Take the equations of motion of 2D dilaton gravity and find their most general solution for the metric g and constant dilaton X. You have to assume that $V(X) = 0$ has at least one real zero in the range of definition of X. A suitable model for testing your results is the Kaluza–Klein reduced gravitational Chern–Simons model with potentials

$$U = 0 \qquad V = \frac{1}{2}\,X(c - X^2)$$

Discuss the geometries of all constant dilaton vacua of this specific model.

These exercises are due on April 26th 2010.

Hints:

- This exercise is analog to the calculations we performed during the lectures. You can choose convenient values for the two integration constants appearing in the definitions

$$Q(X) = Q_0 + \int^X \mathrm{d}y\, U(y) \qquad w(X) = w_0 - 2\int^X \mathrm{d}y\, V(y) e^{Q(y)}$$

  Recall our general solution for the metric in Eddington–Finkelstein coordinates

$$\mathrm{d}s^2 = 2\,\mathrm{d}u\,\mathrm{d}r - K(r)\,\mathrm{d}u^2 \qquad K = e^Q w(1 - \frac{2m}{w})$$

  and for the dilaton

$$\partial_r X = e^{-Q}$$

  The ground state solution arises if you set $m = 0$ (typically for the choice $w_0 = 0$).

- You get the number of Killing horizons simply by counting the number of real zeros in the Killing norm (squared) $K = e^Q w(1 - \frac{2m}{w})$.

- Recall the equations of motion for 2D dilaton gravity from the lectures and exploit $X = \text{const.}$ as early as possible. Exploit that the Ricci scalar R uniquely determines the Riemann tensor in 2D — if you know e.g. that R is constant then spacetime can only be deSitter, Minkowski or Anti-deSitter, depending on the sign of R.