Daniel Grumiller May 3rd 2010

Black Holes II — Exercise sheet 7

(17.1) **First Law**

The first law of black hole mechanics states

$$\mathrm{d}M = \frac{\kappa}{8\pi}\,\mathrm{d}A + \Omega_H\,\mathrm{d}J + \Phi_H\,\mathrm{d}Q$$

where $M, \kappa, A, \Omega_H, J, \Phi_H$ and Q are mass, surface gravity, event horizon area, angular velocity Ω_H, angular momentum J, electric potential Φ_H and electric charge Q of the black hole. "Derive" this result a la Gibbons from the Smarr formula

$$M = \frac{\kappa A}{4\pi} + 2\Omega_H J + \Phi_H Q$$

Compare with the first law of thermodynamics and relate corresponding quantities. Calculate κ for a solar mass Schwarzschild black hole and convert the result into Kelvin.

(17.2) **Second Law**

The second law of black hole mechanics states

$$\mathrm{d}A \geq 0$$

where A is the event horizon area. By comparison with (17.1) A must be proportional to entropy. Calculate A for a solar mass Schwarzschild black hole and provide an estimate of the number of microstates of such a black hole. Discuss (either colloquially or with formulas) what happens when you take a box filled with photons of a certain temperature, energy and entropy and drop it into the black hole.

(17.3) **Third Law**

The third law of black hole mechanics states that physical processes that lead to

$$\kappa \to 0$$

are not possible in finite time. Discuss for a Schwarzschild black hole how you could attempt to violate the third law and why such attempts do not work. Generalize this discussion to Reissner–Nordström black holes.

Note: The three laws above are valid under the assumptions mentioned in the lectures.

These exercises are due on May 10th 2010.

Hints:

- Recall the mass dimensions of A, J and Q^2, and exploit Euler's formula for homogeneous functions. Convert the Q^2-dependence into Q-dependence (you may assume that $Q \geq 0$ with no loss of generality). For the comparison with thermodynamics note that mass and energy are the same physical quantities, and that changing angular momentum or charge corresponds to work terms. Regarding the last part: if you forgot the relation between surface gravity and Killing norm look at exercise (8.2).

- Calculate A in natural units and recall how the number of microstates scales with entropy. For the colloquial discussion compare with exercise (8.3).

- Remember how surface gravity is related to mass and consider what you would have to do with the mass of a Schwarzschild black hole in order to make surface gravity vanish. For the Reissner–Nordström case start with a sub-extremal black hole $|Q| < M$ and try to make it extremal by dropping charged particles into it. Note that the particle only falls into the black hole if gravitational attraction overcomes the electrostatic repulsion.