Daniel Grumiller March 11th 2014

Black Holes II — Exercise sheet 2

(12.1) **Penrose diagram for Robinson–Bertotti**
We derived last semester (see exercise 9.1) the near horizon limit geometry of the extremal Reissner–Nordström black hole, which is known as Robinson–Bertotti geometry. Its line element is given by

$$\mathrm{d}s^2 = -\lambda^2 \, \mathrm{d}t^2 + Q^2 \, \frac{\mathrm{d}\lambda^2}{\lambda^2} + Q^2 \, \mathrm{d}\Omega^2_{S^2}$$

where Q is a constant (the charge) and $\mathrm{d}\Omega^2_{S^2}$ is the line-element of the round S^2. Show that the singularity at $\lambda = 0$ is merely a coordinate singularity. Show further that $\lambda = 0$ is a degenerate Killing horizon with respect to ∂_t. Finally, obtain the maximal analytic extension of the Robinson–Bertotti metric and deduce its Penrose diagram.

(12.2) **Inventing Penrose diagrams**
Draw (2-dimensional) Penrose diagrams for spacetimes with the following properties:

(a) Asymptotically flat, event horizon, no singularity

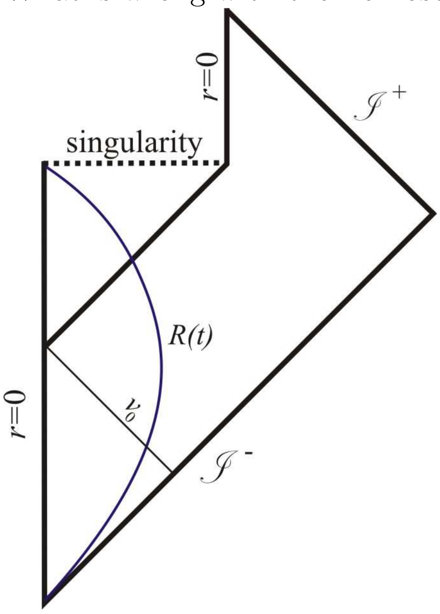
(b) Asymptotically flat, as many Killing horizons as possible, no Cauchy horizon

(c) Asymptotically flat, no event horizon, singularity

(d) Asymptotically flat, two non-extremal and one extremal Killing horizon

(e) Asymptotically flat, at least one Killing horizon, no singularity, no event horizon

(12.3) **Penrose diagram of semi-classically evaporating black hole**
What is wrong with the Penrose diagram below?



These exercises are due on March 18th 2014.

Hints:

- For the first two questions the coordinate transformation $u = t + Q/\lambda$, $v = t - Q/\lambda$ is helpful. For the final task the coordinate transformation $u = \tan(U/2)$, $v = -\cot(V/2)$ is convenient.

- Follow the algorithm explained during the lectures: start with the asymptotically flat region and "design" an Eddington–Finkelstein patch such that all requirements of the sub-exercise are met. Then, if possible, glue together copies of this Eddington–Finkelstein patch (and/or flipped versions thereof).

- Consider the domain of dependence of various $t =$const. hypersurfaces. Are there Cauchy hypersurfaces? Is spacetime globally hyperbolic? (Why) should we care about these questions?