Daniel Grumiller March 25th 2014

Black Holes II — Alternative exercise sheet 4

(14.1) **Second order field equations for 2D dilaton gravity**
Derive the equations of motion by varying the 2D dilaton gravity action

$$S^{\text{2DG}} = \frac{1}{\kappa}\int \mathrm{d}^2x\sqrt{-g}\,\Big[XR - U(X)(\nabla X)^2 - 2V(X)\Big]$$

with respect to the dilaton field X and the metric $g_{\mu\nu}$ (U, V are arbitrary functions of X). You may neglect surface terms in this exercise. Compare your result with Eqs. (2.1) and (2.2) in `hep-th/0703230`.

(14.2) **Constant dilaton vacua**
Take the equations of motion of 2D dilaton gravity and find their most general solution for the metric g and constant dilaton X. You have to assume that $V(X) = 0$ has at least one real zero in the range of definition of X. A suitable model for testing your results is the Kaluza–Klein reduced gravitational Chern–Simons model with potentials

$$U = 0 \qquad V = \frac{1}{2}\,X(c - X^2)$$

Discuss the geometries of all constant dilaton vacua of this specific model.

(14.3) **Jackiw–Teitelboim, Katanaev–Volovich, CGHS et al.**
Consider 2D dilaton gravity (see above) with the following potentials

(i) $U = 0$, $V = -\Lambda X$ (Jackiw–Teitelboim)
(ii) $U = \alpha$, $V = \beta X^2 - \lambda$ (Katanaev–Volovich)
(iii) $U = 0$, $V = -\frac{1}{2}$ (Callan–Giddings–Harvey–Strominger)
(iv) $U = a$, $V = e^{\alpha X}$ (Liouville gravity)
(v) $U = 0$, $V = \frac{1}{2}\,X(c - X^2)$ (Kaluza–Klein reduced Chern–Simons)

Which of these models belong to the ab-family $U(X) = -\frac{a}{X}$, $V(X) = -\frac{1}{2}\,X^{a+b}$? Which of these models have constant dilaton vacua [see exercise (14.2)]? Which of these models have a Minkowski ground state? How many Killing horizons are there for each model?

These alternative exercises are due on April 1st 2014 and can be combined with the other set of exercises (at most three exercises/person can be submitted).

Hints:

- The dilaton variation is straightforward [analog to exercise (3.2c)]. For the metric variation use the formula $\delta\sqrt{-g} = \frac{1}{2}\sqrt{-g}\,g^{\mu\nu}\,\delta g_{\mu\nu}$ (see previous semester). As we showed last semester (see also later this semester) the variation of the Ricci scalar yields

$$\delta R = -R^{\mu\nu}\,\delta g_{\mu\nu} + \nabla^\mu\nabla^\nu\,\delta g_{\mu\nu} - g^{\mu\nu}\nabla^2\,\delta g_{\mu\nu}$$

  Exploit also the fact that the 2D Einstein tensor vanishes identically for any 2D metric, $R_{\mu\nu} = \frac{1}{2}\,g_{\mu\nu}R$. Be careful with signs!

- Recall the equations of motion for 2D dilaton gravity from the lectures and exploit $X =$ const. as early as possible. Exploit that the Ricci scalar R uniquely determines the Riemann tensor in 2D — if you know e.g. that R is constant then spacetime can only be de Sitter, Minkowski or Anti-de Sitter, depending on the sign of R.

- The first three questions should be straightforward to answer. You get the number of Killing horizons simply by counting the number of real zeros in the Killing norm (squared) $K = e^Q w(1 - \frac{2m}{w})$. Recall that the two functions Q and w were defined as $Q(X) = Q_0 + \int^X \mathrm{d}y\,U(y)$ and $w(X) = w_0 - 2\int^X \mathrm{d}y\,V(y)e^{Q(y)}$, respectively.