Prerequisites

Pollack&Stump, Chapter 15, or Zangwill 23

34 The retarded time

A point charge q moves along a specified trajectory $\boldsymbol{r}_0(t)$ with velocity $\boldsymbol{v}(t) = \dot{\boldsymbol{r}}_0(t)$. For each choice of t, show that the equation $t_{\mathrm{R}} = t - |\boldsymbol{r} - \boldsymbol{r}_0(t_{\mathrm{R}})|/c$ has exactly one solution for the retarded time t_{R}, provided $|\boldsymbol{v}(t)| < c$. Proof both, existence and uniqueness.

35 Oscillating dipole

The average power radiated by an oscillating dipole, with dipole moment $p(t) = p_0 \cos \omega t$, is

$$\overline{P} = \frac{p_0^2 \omega^4}{12\pi\epsilon_0 c^3}.$$

Derive this result from the Larmor formula, treating the dipole as an oscillating pair of charges $\pm q_0$, which oscillate 180 degrees out of phase with amplitude of oscillation $d/2$, such that $p_0 = q_0 d$. Hint: The waves radiated by q_0 and $-q_0$ interfere.

36 The classical radiation spectrum of beta decay

Model the beta decay reaction $n \to p + e + \bar{\nu}_e$ as the abrupt creation of an electron at $t = 0$ with constant velocity $\boldsymbol{v} = c\boldsymbol{\beta}$.

(a) Find the angular distribution of energy radiated per unit frequency, $\mathrm{d}I/\mathrm{d}\Omega$. Use θ for the angle between $\boldsymbol{v}$ and the observation point. Hint: Consider the use of a convergence factor if you encounter an ill-behaved integral.

(b) Show that the total energy radiated per unit frequency is

$$I(\omega) = \frac{\mu_0 q^2 c}{4\pi^2}\left[\frac{1}{\beta}\log\left(\frac{1+\beta}{1-\beta}\right) - 2\right].$$

(c) The fact that $\mathrm{d}I/\mathrm{d}\Omega$ and $I(\omega)$ are independent of frequency implies that the total amount of energy radiated is infinite in this model. Assume 10 GeV as an upper frequency limit and estimate the resulting total radiated power.

37 Radiation energy loss from Coulomb repulsion

A non-relativistic particle with charge q, mass m, and initial speed v_0 collides head-on with a fixed field of force. The force is Coulombic with potential $V(r) = Ze/r$. Integrate Larmor's formula to show that the total energy lost by the particle to radiation is $\Delta E = 2mv_0^5/45\pi\epsilon_0 Zc^3$.

38 Radiation of accelerated electrons (computer exercise)

Suppose the velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ of a charged particle are orthogonal. Let $\mathrm{d}P/\mathrm{d}\Omega$ be the differential power radiated. Call the total power $P_T = \int(\mathrm{d}P/\mathrm{d}\Omega)\mathrm{d}\Omega$. Use computer graphics to plot the angular distribution of the power

$$f(\theta) = \frac{1}{P_T}\frac{\mathrm{d}P}{\mathrm{d}\Omega},$$

as a function of the angle θ with the direction of motion, for these speeds: $0.01\,c$, $0.5\,c$, and $0.99\,c$. Normalize your plots such that they all show the same angular intensity in the direction of motion.