5. Exercise of QFT for many-body systems

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12. "Absorption" and response functions *2+1.5=3.5 points*

Consider the following (external) perturbation term $v(t)$ to the Hamiltonian $\mathcal{H}$ of a given physical system:

$$v(t) = -\sum_i a_i(t) A_i.$$

so that the total Hamiltonian (system+ ext. perturbation) is $\mathcal{H}_t = \mathcal{H} + v(t)$.

a) From the Heisenberg equation of motion for $\mathcal{H}$, calculate the energy absorption rate for the given system

$$\left\langle \frac{d}{dt}\mathcal{H} \right\rangle_v \tag{1}$$

up to the second order in the external perturbation $v(t)$.
Hint: Apply the "Kubo (Nakano) formula" to $\langle[\mathcal{H}, A_i]\rangle$ and use the Heisenberg equation of motion to rewrite $[\mathcal{H}, A_i(t)]$ as time derivative of $A_i(t)$. It is also useful to introduce a susceptibility defined as

$$\chi_{ij}(t-t') = -\frac{1}{i\hbar}\Theta(t-t')\langle[A_i(t), A_j(t')]\rangle_{v=0}$$

b) In the case of a monochromatic perturbation, namely $a_j(t) = \frac{1}{2}\left(a_j \mathrm{e}^{-i\omega t} + a_j^* \mathrm{e}^{i\omega t}\right)$ show that the temporal average of the energy absorption rate in Eq. (1) is related to the imaginary part of $\chi_{ij}(\omega)$.

13. Magnetic susceptibilities in d=2 *1.5+1.5+1.5+2*=4.5 + 2* points*

Consider a system of non-interacting electrons on a square lattice (with lattice spacing $a = 1$) in two dimensions at half-filling (particle-hole symmetric case), whose energy dispersion is given by

$$\varepsilon_{\mathbf{k}} = -2t\left(\cos k_x + \cos k_y\right).$$

a) Expand $\varepsilon_{\mathbf{k}}$ around the saddle-point S $= (\pi, 0)$ and calculate the corresponding density of states around the saddle-point energy.

b) Compute the ferromagnetic susceptibility, i.e. the Fourier transform of the Spin-Spin response function $\langle T_\tau S_z(\mathbf{r}_i, \tau) S_z(0,0)\rangle$ for a momentum $\mathbf{Q} = (0,0)$ and a frequency $\Omega_m = 0$, and, on the basis of the results of **13 a)**, estimate the leading diverging term in the limit of $T \to 0$.

c) Compute the antiferromagnetic susceptibility, i.e. the Fourier transform of the Spin-Spin response function $\langle T_\tau S_z(\mathbf{r}_i, \tau) S_z(0,0) \rangle$ for a momentum $\mathbf{Q} = (\pi, \pi)$ and a frequency $\Omega_m = 0$, and, on the basis of the results of **13 a)**, estimate the leading diverging term in the limit $T \to 0$.

d) Consider the same electronic system, but now in presence of a local (repulsive) Hubbard interaction $U > 0$, and calculate within the random-phase approximation (RPA) the two magnetic susceptibilities of **13b)** and **13c)** in the corresponding particle-hole channel. On the basis of your RPA calculations, make your final considerations about the tendency of the system towards a given magnetic order at $T = 0$.

* Bonus points

Viel Spaß!