IFP
TU Wien

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8. Übung QFT für Vielteilchen-Systeme

28.06.2012, 14:00-16:00, Seminarraum 138C

1. Feynman Diagram “quiz” *1+0.5+1=2.5 Punkte*

Consider the following eight Feynman diagrams (for the Green’s function of an interacting electronic system):

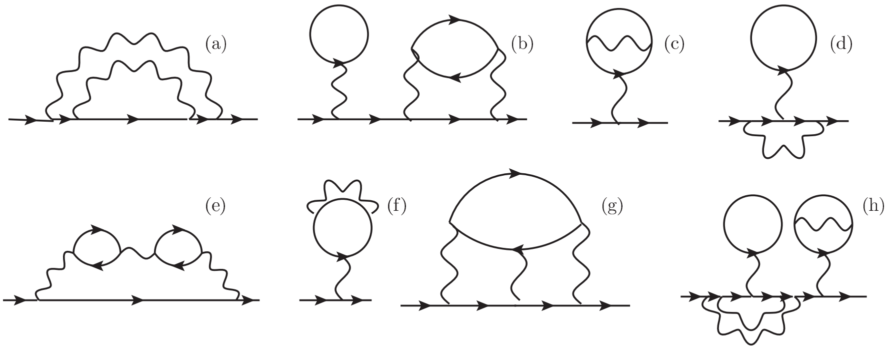

Figure 1: eight Feynman diagrams of second and higher orders

a) Classify the eight diagrams as *reducible* (also defined as “type A” in the Lecture) or *irreducible*, specifying if they are *non-skeleton* (“type B”), or *skeleton** (“type C”) diagrams. Then, draw a new irreducible skeleton diagram of third order different from any of those appearing in Fig. 1.

b) Are any of the diagrams shown in Fig. 1 topologically equivalent? If yes, which ones?

c) Calculate the numerical prefactor of all the eight diagrams shown in Fig. 1, according to the standard Feynman rules.

* As suggested by their name, the “skeleton” diagrams are diagrams, which do not contain any self-energy insertion in the internal lines (“type C”) diagrams.

2. Magnetic susceptibilities in d=2 *1.5+1.5+1.5+2*=4.5 + 2* Punkte*

Consider a system of non-interacting electrons on a square lattice (with lattice spacing $a = 1$) in two dimensions at half-filling (particle-hole symmetric case), whose energy dispersion is given by

$$\varepsilon_{\mathbf{k}} = -2t\left(\cos k_x + \cos k_y\right).$$

a) Expand $\varepsilon_{\mathbf{k}}$ around the saddle-point $\mathbf{k_s} = (\pi, 0)$ and calculate the corresponding density of states around the saddle-point energy.

b) Compute the <u>ferromagnetic</u> susceptibility, i.e. the Fourier transform of the Spin-Spin response function $\overline{\langle T_\tau S_z(\mathbf{r}_i,\tau)}S_z(0,0)\rangle$ for a momentum $\mathbf{Q} = (0,0)$ and a frequency $\Omega_m = 0$, and, on the basis of the results of **13 a)**, estimate the leading diverging term in the limit of $T \to 0$.

c) Compute the antiferromagnetic susceptibility, i.e. the Fourier transform of the Spin-Spin response function $\langle T_\tau S_z(\mathbf{r}_i, \tau) S_z(0,0) \rangle$ for a momentum $\mathbf{Q} = (\pi, \pi)$ and a frequency $\Omega_m = 0$, and, on the basis of the results of **13 a)**, estimate the leading diverging term in the limit $T \to 0$.

d) Consider the same electronic system, but now in presence of a local (repulsive) Hubbard interaction $U > 0$, and calculate within the random-phase approximation (RPA) the two magnetic susceptibilities of **13b)** and **13c)** in the corresponding (particle-hole) channel. On the basis of your RPA calculations, make your final considerations about the tendency of the system towards a given magnetic order at $T = 0$.

* Bonus points

Viel Spaß!