IFP
TU Wien

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5. Exercise on QFT for many-body systems

10/06/2016

9. Feynman diagram quiz

1.5+1+1.5=4 points

Consider the following eight Feynman diagrams (for the Green's function of an interacting electronic system):

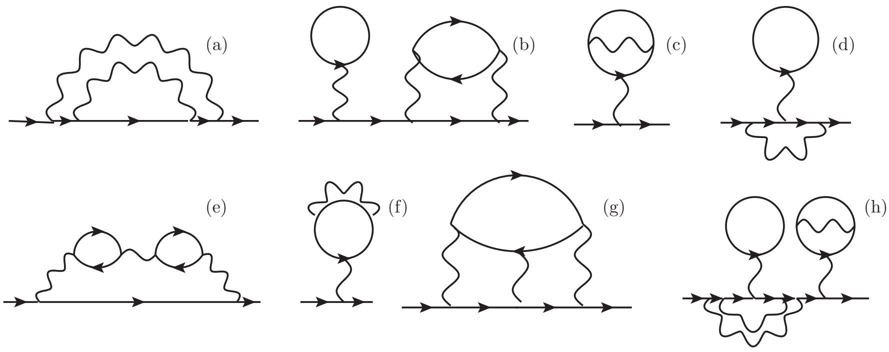

Figure 1: Eight Feynman diagrams of second and higher orders

a) Classify the eight diagrams as *reducible* (they will be defined as "type A" in the Lecture on 19/05/2016) or *irreducible*, specifying if they are *non-skeleton* ("type B"), or *skeleton* ("type C") diagrams[1]. Afterwards draw a new irreducible skeleton diagram of third order different from any of those appearing in Fig. 1.

b) Are any of the diagrams shown in Fig. 1 topologically equivalent? If yes, which ones?

c) Calculate the numerical prefactor of all the eight diagrams shown in Fig. 1, according to the standard Feynman rules.

10. Second-order self-energy diagram

2.5+2.5+1+2=6+2* points*

a) Write the explicit expression of the second-order self-energy diagram shown in Fig. 2 at $T \neq 0$ in terms of the Green's functions on the Matsubara axis.

b) Evaluate the diagram by performing the two internal Matsubara sums. Discuss what is the difference between considering a generic two-particle interaction $\mathcal{H}_V = \frac{1}{2L^d} \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}\sigma\sigma'} V(\mathbf{q}) c^\dagger_{\mathbf{k}+\mathbf{q}\sigma} c^\dagger_{\mathbf{k}'-\mathbf{q}\sigma'} c_{\mathbf{k}'\sigma'} c_{\mathbf{k}\sigma}$ and a local Hubbard interaction of the form $\mathcal{H}_V = U \sum_i n_{i\uparrow} n_{i\downarrow}$ where the sum over i runs over all lattice sites and $n_{i\sigma} = c^\dagger_{i\sigma} c_{i\sigma}$.

[1] As suggested by their name, the "skeleton" diagrams are diagrams, which do not contain any self-energy insertion in the internal lines ("type C" diagrams).

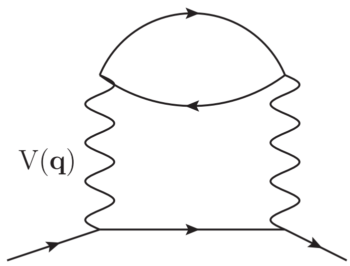

Figure 2: A second order diagram for the self-energy, $\Sigma^{(2)}(\mathbf{k}, i\omega_n)$. For the calculation consider that the incoming line has a definite spin, say $\sigma = \uparrow$.

c) Give a possible physical interpretation of the diagram.

Hint: this diagram may be seen as the first one of a specific "series" of diagrams, whose second term is the diagram (e) of Fig. 1.

d) Calculate the imaginary part of the diagram on the real axis (in the simpler case of a purely local Hubbard interaction U). From the low-T and small-ω limit of this quantity one can provide an estimate of the quasiparticle lifetime. Try to make such estimation, relating your results to the Landau Fermi liquid theory. In particular, identify the contributions to the scattering of electron-like and of hole-like quasiparticles and determine the frequency dependence of Im $\Sigma^{(2)}$ in the low-T and small-ω limit.

Hint: Rearrange the Fermi and Bose functions in such a way that the scattering process can be described by two terms one of which can be obtained from the other by simply reverting all momenta involved in the scattering process (particle-hole transformation).

* Bonus points

Viel Spaß!