4. Exercise on QFT for many-body systems

Sommersemester 2018

TUTORIUM: Friday, 18.05.2018.

7. Linked-cluster theorem *2.5 points*

As it has already been discussed in the Lecture, only *connected* Feynman diagrams have to be considered when calculating the one-particle Green's function. Starting from the perturbation expansion of the Green's function at $T > 0$, show that the time-ordered average for a given order n of perturbation theory decomposes into a product of connected and disconnected diagrams. Prove that the disconnected factor cancels exactly the denominator $Z = \langle S(\beta)\rangle_0$.

Hint: Consider the n-th order term in the perturbation expansion of the numerator of the Green function ($\sim \langle T\, c(\tau)c^\dagger(0)H_V(\tau_1)\cdots H_V(\tau_n)\rangle_0$*). According to Wick's theorem this can be written in terms of connected (*$\sim \langle T\, c(\tau)c^\dagger(0)H_V(\tau_1)\cdots H_V(\tau_m)\rangle_0$*) and disconnected (*$\sim \langle T\, H_V(\tau_{m+1})\cdots H_V(\tau_n)\rangle_0$*) contractions, with* $m = 1,\cdots,n$*.*

8. Feynman diagram quiz *1+0.5+1=2.5 points*

Consider the following eight Feynman diagrams (for the Green's function of an interacting electronic system):

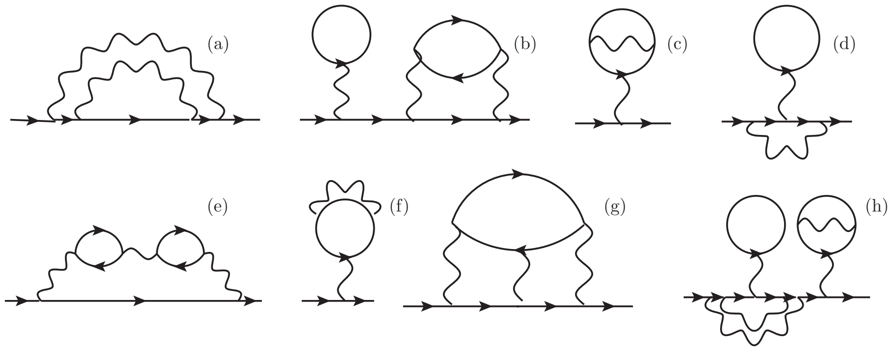

Figure 1: Eight Feynman diagrams of second and higher orders

a) Classify the eight diagrams as *reducible* or *irreducible*, specifying if they are *non-skeleton*, or *skeleton* diagrams[1]. Afterwards draw a new irreducible skeleton diagram of third order different from any of those appearing in Fig. 1.

b) Are any of the diagrams shown in Fig. 1 topologically equivalent? If yes, which ones?

c) Calculate the numerical prefactor of all the eight diagrams shown in Fig. 1, according to the standard Feynman rules.

[1] As suggested by their name, the "skeleton" diagrams are diagrams, which do not contain any self-energy insertion in the internal lines.

9. Second-order self-energy diagram

2+2+1+2.5=5+2.5* points*

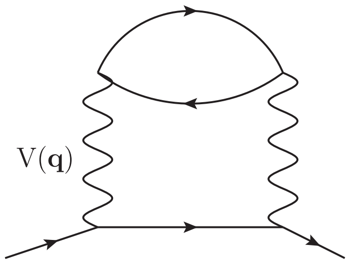

Figure 2: A second order diagram for the self-energy, $\Sigma^{(2)}(\mathbf{k}, i\omega_n)$. For the calculation consider that the incoming line has a definite spin, say $\sigma = \uparrow$.

a) Write the explicit expression of the second-order self-energy diagram shown in Fig. 2 at $T > 0$ in terms of the Green functions on the Matsubara axis.

b) Evaluate the diagram by performing the two internal Matsubara sums. Discuss the difference between considering a generic two-particle interaction $\mathcal{H}_V = \frac{1}{2L^d}\sum_{\mathbf{k}\mathbf{k}'\mathbf{q}\sigma\sigma'} V(\mathbf{q}) c^\dagger_{\mathbf{k}+\mathbf{q}\sigma} c^\dagger_{\mathbf{k}'-\mathbf{q}\sigma'} c_{\mathbf{k}'\sigma'} c_{\mathbf{k}\sigma}$ and a local Hubbard interaction of the form $\mathcal{H}_V = U\sum_i n_{i\uparrow} n_{i\downarrow}$ where the sum over i runs over all lattice sites and $n_{i\sigma} = c^\dagger_{i\sigma} c_{i\sigma}$.

c) Give a possible physical interpretation of the diagram.

Hint: this diagram may be seen as the first one of a specific "series" of diagrams, whose second term is the diagram (e) of Fig. 1.

d) (Bonus points) Calculate the imaginary part of the diagram on the real axis (in the case of the Hubbard interaction). From the low-T and small-ω limit of this quantity one can provide an estimate of the quasiparticle lifetime. Determine the frequency dependence of Im $\Sigma^{(2)}$ in the low-T and small-ω limit.

Remark: The Fermi and Bose functions can be rearanged in such a way that the scattering process can be described by two terms one of which can be obtained from the other by simply reverting all momenta involved in the scattering process (particle-hole transformation). In this way one can identify the contributions to the scattering of electron-like and of hole-like quasiparticles.