Green's Function for a noninteracting system from definition (or Lehmann representation): 1

$\boxed{\tau > 0}$

$$G^0_\sigma(\bar{k},\tau) = -\frac{1}{Z} Tr\left[e^{-\beta H} c_{\bar{k}\sigma}(\tau) c^+_{\bar{k}\sigma}\right] =$$

$$\Big\{ H = \sum_{\bar{k}\sigma} \varepsilon_{\bar{k}} c^+_{\bar{k}\sigma} c_{\bar{k}\sigma}$$

$$= -\frac{1}{Z}\sum_n \langle n| e^{-\beta H} e^{\tau H} c_{\bar{k}\sigma} e^{-\tau H} c^+_{\bar{k}\sigma} |n\rangle =$$

$$= -\frac{1}{Z}\sum_{nm} e^{-\beta E_n} e^{\tau E_n} \langle n| c_{\bar{k}\sigma} |m\rangle e^{-\tau E_m} \langle m| c^+_{\bar{k}\sigma} |n\rangle =$$

$$= -\frac{1}{Z}\sum_{nm} e^{-\beta E_n} e^{\tau(E_n - E_m)} \langle n| c_{\bar{k}\sigma} |m\rangle\langle m| c^+_{\bar{k}\sigma} |n\rangle$$

What are the states $|m\rangle$ and $|n\rangle$?
They are eigenstates of H. The energies?

$$H|m\rangle = \sum_{\bar{k}'\sigma} \varepsilon_{\bar{k}'} n_{\bar{k}'\sigma} |m\rangle = E_m |m\rangle$$

this number tells ~~me~~ how many electrons are there in the state $|m\rangle$ with quantum numbers $\bar{k}'$ and σ:

$$n_{\bar{k}'\sigma} = \begin{cases} 0 \\ 1 \end{cases}$$

The matrix element $\langle m | c^{\dagger}_{\bar{k}\sigma} | n \rangle$ is only nonzero when $|m\rangle$ has one electron more than $|n\rangle$. Exactly the one with $\bar{k},\sigma$. That means $E_m - E_n = \varepsilon_{\bar{k}}$ and we can simplify the sum to give:

$$G^{0}_{\sigma}(\bar{k},\tau) = -\frac{1}{Z} e^{-\tau \varepsilon_{\bar{k}}} \sum_n e^{-\beta E_n} \sum_m \langle n | c_{\bar{k}\sigma} |m\rangle \underbrace{\langle m | c^{\dagger}_{\bar{k}\sigma} | n\rangle}_{1} =$$

$$= -\frac{1}{Z} e^{-\tau \varepsilon_{\bar{k}}} \sum_n e^{-\beta E_n} \langle n | c_{\bar{k}\sigma} c^{\dagger}_{\bar{k}\sigma} | n \rangle =$$

Jetzt wir benutzen die definition:

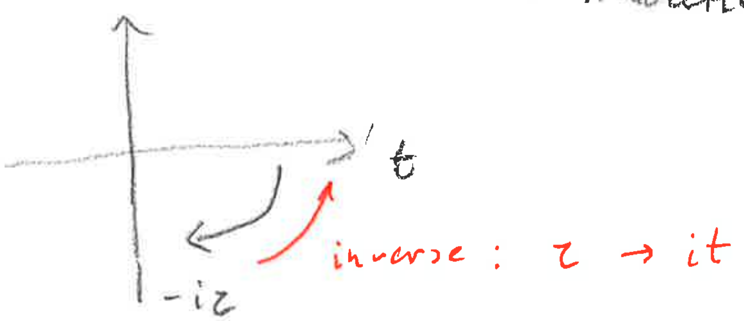
$$\langle n_{\bar{k}\sigma} \rangle = -\frac{1}{Z} \sum_n e^{-\beta E_n} \langle n | c^{\dagger}_{\bar{k}\sigma} c_{\bar{k}\sigma} | n \rangle$$

und $c_{\bar{k}\sigma} c^{\dagger}_{\bar{k}\sigma} = 1 - c^{\dagger}_{\bar{k}\sigma} c_{\bar{k}\sigma}$

$$= -e^{-\tau \varepsilon_{\bar{k}}} \left(1 - \langle n_{\bar{k}\sigma} \rangle\right) = -e^{-\tau \varepsilon_{\bar{k}}} \left(1 - f(\varepsilon_{\bar{k}})\right)$$

Fermi-Dirac distribution

Wick rotation:



convention: additional „i"

$$G^{0}_{\sigma}(\bar{k},t) = -i\, e^{-it \varepsilon_{\bar{k}}} \left(1 - f(\varepsilon_{\bar{k}})\right) \quad \text{for } t > 0$$

free propagator of a particle with $\varepsilon_{\bar{k}\sigma}$