4. Exercise on QFT for many-body systems

Sommersemester 2023

<u>TUTORIUM</u>: Friday, 26.05.2023.

7. Linked-cluster theorem *2.5 points*

As it has already been discussed in the Lecture, only *connected* Feynman diagrams have to be considered when calculating the one-particle Green's function. Starting from the perturbation expansion of the Green's function at $T > 0$, show that the time-ordered average for a given order n of perturbation theory decomposes into a product of connected and disconnected diagrams. Prove that the disconnected factor cancels exactly the denominator $Z = \langle S(\beta)\rangle_0$.

Hint: Consider the n-th order term in the perturbation expansion of the numerator of the Green function ($\sim \langle T\, c(\tau)c^\dagger(0)H_V(\tau_1)\cdots H_V(\tau_n)\rangle_0$*). According to Wick's theorem this can be written in terms of connected (*$\sim \langle T\, c(\tau)c^\dagger(0)H_V(\tau_1)\cdots H_V(\tau_m)\rangle_0$*) and disconnected (*$\sim \langle T\, H_V(\tau_{m+1})\cdots H_V(\tau_n)\rangle_0$*) contractions, with* $m = 1,\cdots,n$.

8. Feynman diagrams *1+1=2 points*

a) Draw all topologically inequivalent second-order diagrams for the Green's function. Determine the numerical prefactors of all these diagrams according to the standard Feynman rules. Which of these diagrams can be split in two by cutting one Green's function line?

b) Draw 5 topologically inequivalent third-order diagrams for the Green's function. Determine the numerical prefactors of these diagrams according to the standard Feynman rules. Which of the 5 diagrams you have drawn can be split in two by cutting one Green's function line?

9. Second-order diagram *2+3+0.5=5.5 points*

a) Write the explicit expression of the second-order diagram shown in Fig. 1 at $T > 0$ in terms of the Green's functions on the Matsubara axis.

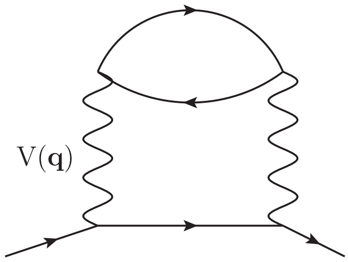

Figure 1: A second order diagram for the Green's function, $G^{(2)}(\mathbf{k}, i\omega_n)$. For the calculation consider that the incoming line has a definite spin, say $\sigma = \uparrow$.

b) Evaluate the diagram by performing the two internal Matsubara sums. Discuss the difference between considering a generic two-particle interaction $\mathcal{H}_V = \frac{1}{2L^d} \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}\sigma\sigma'} V(\mathbf{q}) c^\dagger_{\mathbf{k}+\mathbf{q}\sigma} c^\dagger_{\mathbf{k}'-\mathbf{q}\sigma'} c_{\mathbf{k}'\sigma'} c_{\mathbf{k}\sigma}$ and a local Hubbard interaction of the form $\mathcal{H}_V = U \sum_i n_{i\uparrow} n_{i\downarrow}$ where the sum over i runs over all lattice sites and $n_{i\sigma} = c^\dagger_{i\sigma} c_{i\sigma}$.

c) Give a possible physical interpretation of the diagram.

Hint: this diagram may be seen as the first one of a specific "series" of diagrams.