Exercise 28/29.04.2020

Poisson's equation in 1-dimension

The three-dimensional Poisson equation for electrostatics is a partial differential equation of elliptic type which describes the potential field of a charge density distribution. In SI units, it reads as

$$\nabla^2\phi(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon} \tag{1}$$

where $\phi(\vec{r})$ is the electrostatic potential generated by the charge density $\rho(\vec{r})$ and ϵ is the dielectric permittivity of the medium.

1. Consider the one-dimensional Poisson equation and pursue an approximate solution by the finite difference method: discretize the equation, remember the three-points approximation for the second derivative and write down the resulting set of linear equations in the matrix form $D\vec{u} = \vec{f}$, where D is a square matrix, $\vec{f}$ is the source term and $\vec{u}$ is the unknown term. Which kind of matrix is D?

2. Solve the system with a source term $f(x) = 100\mathrm{e}^{-10x}$, $0 \leq x \leq 1$ and boundary conditions $u(0) = u(1) = 0$. To solve the system, use both the Gauss elimination method and the Thomas algorithm, calculate their respective CPU-times on varying the number n of grid points and report the results in a short table. Which is the most efficient algorithm and why?

3. Plot the numerical solution(s) against the exact solution

$$u(x) = 1 - (1 - \mathrm{e}^{-10})x - \mathrm{e}^{-10x} \tag{2}$$

   for $n = 10$ and 100.

4. Use the LAPACK subroutine DPTSV to solve the system, compare its solution and CPU-time to the results of your self-made code.

Instructions

During the exercise a short protocol must be made and saved as `PROTOKOLL.txt` in the directory of the respective exercise day. The protocol is a simple ASCII text file that is created with a text editor with which you can also write your programs. The protocol must contain the following

1. Date, exercise number, group number, name(s) of the participating students

2. Time required for the tasks (approximately)
3. Name of the created files, the files must be located in the directory of the respective exercise day
4. The answers to any questions asked above
5. Possible problems or peculiarities, if they have occurred.

Note: You should get a plot like this one.