Exercise 13.04.2021

Consider a system of n carts connected by springs, as shown in Figure 1, where the i-th spring has stiffness k_i and each cart is subjected to a constant force f_i. Let x_i denote the amount by which carts i moves when the forces are applied. For each cart i the new equilibrium position is that point at which the sum of the forces on the cart is zero $-k_i(x_i - x_{i-1}) + k_{i+1}(x_{i+1} - x_i) + f_i = 0$.

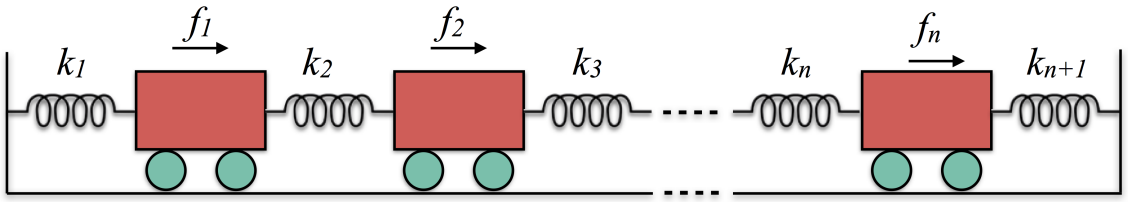

Figure 1: System of n carts connected by springs.

- Write the related system of n linear equations in the matrix form as $A\vec{x} = \vec{f}$. Note that if n is large, the vast majority of the elements of A are zeros; matrices with this property are called *sparse*. Since all of the non-zero elements are confined to a narrow band around the main diagonal, A is also called *banded*. In particular, as the non-zeros are confined to three diagonals, A is *tridiagonal*.

- Now consider the case with $n = 5$ where all $k_i = 4\text{N/m}$ and $f_i = i\text{N}$. To solve the system, use (i) the Gauss elimination method, (ii) the Thomas algorithm, and (iii) the LAPACK subroutine DPTSV.

- Compare the components of the displacement vector obtained with the three different methods. If you plot, *e.g.*, the x_i calculated with the Thomas algorithm or by means of the Lapack subroutine versus the x_i calculated with the Gauss elimination method, all points should fall along the plane bisector, as shown in Figure 2.

- Now calculate the CPU-time of the Gauss method versus the CPU-time of the Thomas algorithm for, *e.g.*, $n = 1000$: which is the most efficient algorithm and why?

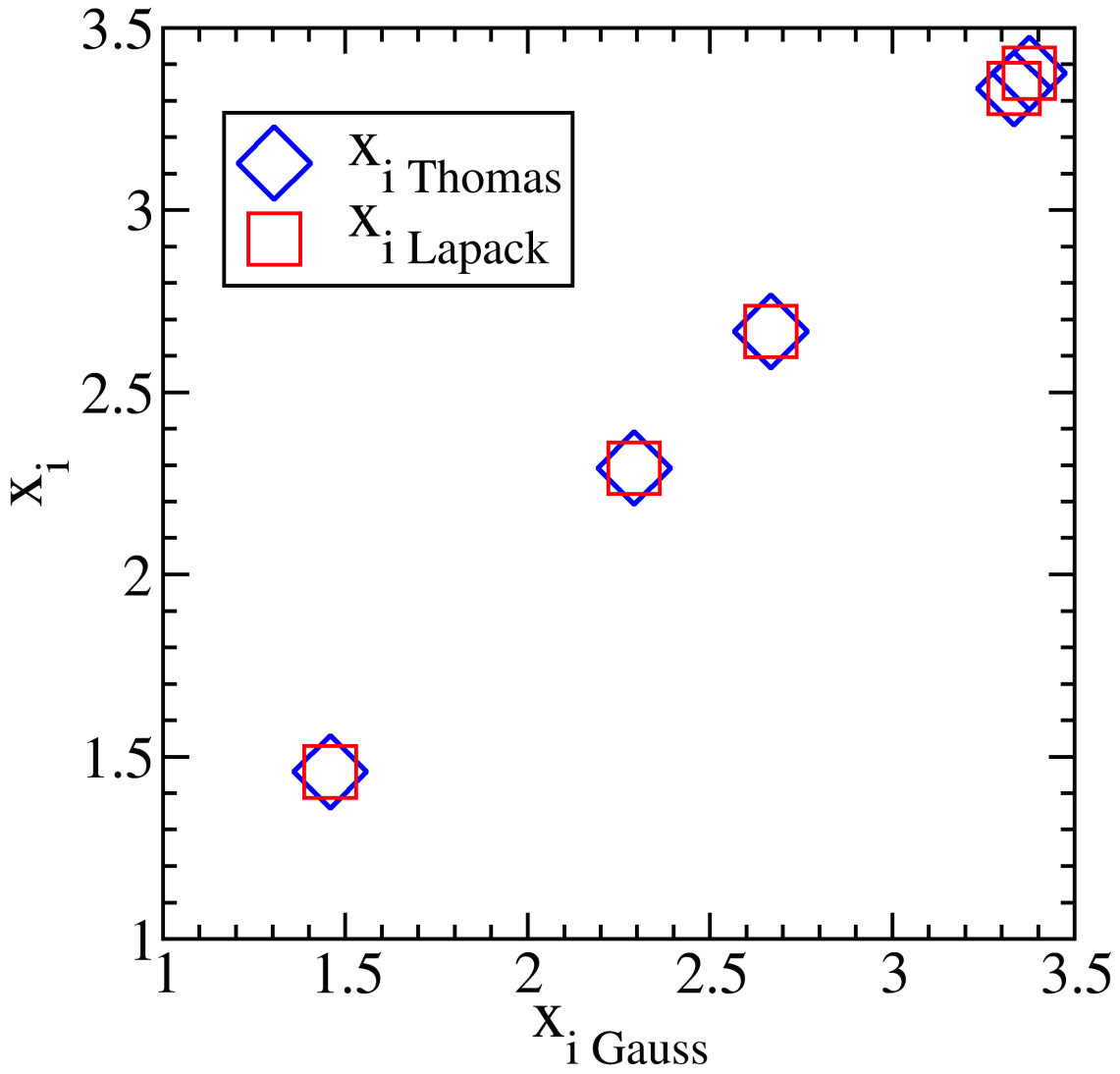

Figure 2: Comparison between the components of the displacement vector calculated with the Gauss elimination method, the Thomas algorithm and the DPTSV LAPACK subroutine for a system of $n = 5$ carts connected by springs with $k_i = 4\text{N/m}$ and subjected to the external forces $f_i = i\text{N}$, as shown in Figure 1.

Instructions

During the exercise a short protocol must be made and saved as `PROTOKOLL.txt` in the directory of the respective exercise day. The protocol is a simple ASCII text file that is created with a text editor with which you can also write your programs. The protocol must contain the following

1. Date, exercise number, group number, name(s) of the participating students
2. Time required for the tasks (approximately)
3. Name of the created files, the files must be located in the directory of the respective exercise day
4. The answers to any questions asked above
5. Possible problems or peculiarities, if they have occurred.