6. Consider QED as Abelian gauge theory. The photon field fulfils the transformation property

$$V'_\mu = V_\mu + \frac{1}{g} \partial_\mu \lambda(x). \tag{7}$$

Write down how a mass term for the photon field should look like and check whether it is gauge invariant then.

7. The kinetic term for the gauge field in QED is

$$\mathcal{L}_V = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{8}$$

Show its local gauge invariance.

8. Given is the group SU(3) with

$$[T_a, T_b] = i\, f_{abc} T_c\,, \tag{9}$$

and the covariant derivatives

$$D_\mu = \partial_\mu + ig T_a G^a_\mu\,. \tag{10}$$

In order to make the free Dirac Lagrange density $\mathcal{L}_0 = \bar{\Psi}(i\slashed{\partial} - m)\Psi$ gauge invariant under SU(3) use D_μ instead of ∂_μ. Check whether the resulted Lagrange density $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$ with the analogous to QED interaction term

$$\mathcal{L}_I = -g \left(\bar{\psi} \gamma^\mu T_a \psi \right) G^a_\mu \tag{11}$$

is gauge invariant under the transformations

$$\psi' = (1 + i\alpha^a T_a)\, \psi \tag{12}$$

for the matter field and the QED analogous transformation for the gauge field,

$$G^{a\prime}_\mu = G^a_\mu - \frac{1}{g} (\partial_\mu \alpha^a)\,. \tag{13}$$

What can be done to cure this?

9. Show the invariance of the kinetic term of the gauge field under an infinitesimal non-Abelian local transformation with

$$U(x) = e^{-ig\alpha_a T^a}. \tag{14}$$

Calculate the infinitesimal transformation behaviour of $F_{\mu\nu}$ and of the gauge field.