8. Given is the group SU(3) with

$$[T_a, T_b] = i\, f_{abc} T_c\,, \tag{9}$$

and the covariant derivatives

$$D_\mu = \partial_\mu + igT_a G^a_\mu\,. \tag{10}$$

In order to make the free Dirac Lagrange density $\mathcal{L}_0 = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi$ gauge invariant under SU(3) use D_μ instead of ∂_μ. Check whether the resulted Lagrange density $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$ with the analogous to QED interaction term

$$\mathcal{L}_I = -g\left(\bar{\psi}\gamma^\mu T_a \psi\right) G^a_\mu \tag{11}$$

is gauge invariant under the transformations

$$\psi' = (1 + i\alpha^a T_a)\,\psi \tag{12}$$

for the matter field and the QED analogous transformation for the gauge field,

$$G^{a\,\prime}_\mu = G^a_\mu - \frac{1}{g}\left(\partial_\mu \alpha^a\right)\,. \tag{13}$$

What can be done to cure this?

9. Show the invariance of the kinetic term of the gauge field under an infinitesimal non-Abelian local transformation with

$$U(x) = e^{-ig\alpha_a T^a}. \tag{14}$$

Calculate the infinitesimal transformation behaviour of $F_{\mu\nu}$ and of the gauge field.