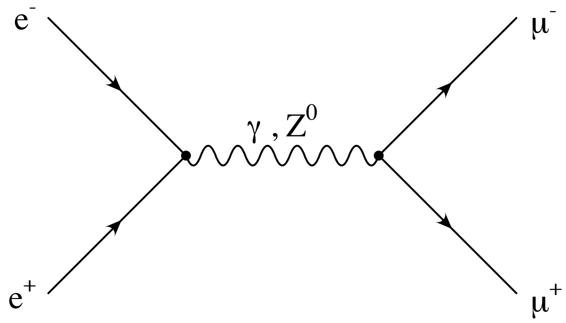
16. Given is the reaction $e^+ + e^- \to \mu^+ + \mu^-$. This reaction can be mediated via the electromagnetic and the weak force carrier γ and Z^0.



First calculate $\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$, where. $\cos\theta$ is the angle between incoming e^- and outgoing μ^- in the CMS. The masses of electron and muon are neglected and the corresponding Feynman rules can be found in the lecture notes.

Then calculate the function

$$A_{FB} = \frac{\int_0^1 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\,\mathrm{d}\cos\theta - \int_{-1}^0 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\,\mathrm{d}\cos\theta}{\int_0^1 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\,\mathrm{d}\cos\theta + \int_{-1}^0 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\,\mathrm{d}\cos\theta}\,. \tag{46}$$

A_{FB} is the abbreviation for "Asymmetry–Forward–Backwards". Verify that

$$A_{FB}(\sqrt{s} = m_Z) = 3(1 - 4\sin^2\theta_W)^2\,. \tag{47}$$

This measurement leads to a very accurate determination of the Weinberg angle. In order to be able to evaluate A_{FB} also directly at the Z-pole, (47), we include the finite
Z-width $\Gamma_Z(\leq m_Z)$ in the denominator of the Z-propagator, $s - m_Z^2 \to s - m_Z^2 + i\,m_Z\Gamma_Z$.