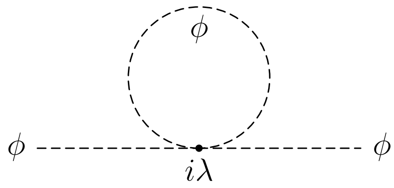
43. Working in dimensional regularisation as regularisation scheme in D-dimensions, in every closed loop with a four-particle vertex there occurs the so-called one-point function A_0, with e.g. the matrix element $\mathcal{M} = i\lambda A_0(m)$, depicted by the Feynman graph



It is defined as

$$A_0\left(m\right) = \frac{\left(2\pi\mu\right)^{4-D}}{i\pi^2}\int d^D k \frac{1}{k^2 - m^2 + i\epsilon}\,. \tag{57}$$

Using the Wick rotation $k^0 \to ik_E^0$ the integration path in the complex k^0 plane will be rotated by $\pi/2$ in order to get Euklidian coordinates. Use the Bogolubov-Schwinger parametrisation

$$\frac{1}{A} = \int_0^\infty dx\, e^{-xA} \qquad A > 0\,. \tag{58}$$

Show that the intermediate result has the form

$$\frac{i}{16\pi^2} A_0\left(m\right) = -i\frac{\mu^{4-D}}{\left(2\pi\right)^D}\int_0^\infty d\alpha\, e^{-\alpha m^2}\int d^D\, k_E\, e^{-\alpha k_E^2}\,. \tag{59}$$

For the integration over the D-dimensional momentum space work with spherical coordinates. For that step these formulas are helpful:

$$\int d\Omega_D = \frac{2\pi^{D/2}}{\Gamma\left(D/2\right)} \quad \text{and} \quad \int_0^\infty dx\, x^a\, e^{-bx^2} = \frac{\Gamma\left(\frac{a+1}{2}\right)}{2b^{\frac{a+1}{2}}} \qquad b > 0, \qquad a \in N_u\,. \tag{60}$$

The remaining parameter integral is partially integrated (the exponent of α becomes $1 - D/2$). By using

$$\int_0^\infty dx\, x^a\, e^{-bx} = \Gamma\left(a+1\right) b^{-a-1} \tag{61}$$

and also $\epsilon = 4 - D$ verify the result

$$\frac{i}{16\pi^2} A_0\left(m\right) = \frac{2im^2}{\left(4\pi\right)^2}\frac{\Gamma\left(\frac{\epsilon}{2}\right)}{2-\epsilon}\, e^{\frac{\epsilon}{2}\left(\ln 4\pi + \ln\frac{\mu^2}{m^2}\right)}\,. \tag{62}$$

Perform an expension in ϵ, ($\Gamma\left(\frac{\epsilon}{2}\right) = \frac{2}{\epsilon} - \gamma + \mathcal{O}\left(\epsilon\right)$)[1] and show that the final result is

$$A_0\left(m\right) = m^2\left(\frac{2}{\epsilon} - \gamma + 1 + \ln 4\pi + \ln\frac{\mu^2}{m^2}\right) + \mathcal{O}\left(\epsilon\right)\,. \tag{63}$$

[1] $\gamma = 0.5772157\ldots$ Euler-Mascheroni constant