**39)** Berechnen Sie mit Hilfe der Protonenwellenfunktion $|p\uparrow\rangle$ und der magnetischen Momente der up- und down-quarks (siehe Anhang)

**a)** das magnetische Moment des Protons $\mu_p$

**b)** das magnetische Moment des Neutrons $\mu_n$ und daraus das Verhältnis $\frac{\mu_p}{\mu_n}$

**40)** Berechnen Sie, unter Berücksichtigung der Isospinerhaltung in der starken Wechselwirkung, mit Hilfe der Isospin-Wellenfunktionen $\phi(I, I_3)$, die folgenden Zerfallsraten:

**a)** $\Gamma(\Delta^- \to \pi^- n)$

**b)** $\Gamma(\Delta^0 \to \pi^- p)$

**c)** $\Gamma(\Delta^0 \to \pi^0 n)$

**d)** $\Gamma(\Delta^+ \to \pi^+ n)$

**e)** $\Gamma(\Delta^+ \to \pi^0 p)$

**f)** $\Gamma(\Delta^{++} \to \pi^+ p)$

Bestimmen Sie abschließend die relativen Häufigkeiten der einzelnen Zerfälle.

quarks as they move within and interact with the QCD potential inside baryons and mesons. Since the QCD environments within baryons and mesons will be different, it should not be a surprise that the constituent masses are different for baryons and mesons. This distinction between current and constituent quark masses implies that only 1% of the mass of a proton is attributable to the masses of the quarks, the remainder arises from the energy associated with the internal QCD gluon field.

### 9.6.5 Baryon magnetic moments

In Chapter 7 it was seen that the magnetic moment of the proton differs from that expected for a point-like Dirac fermion. The experimentally measured values of the anomalous magnetic moments of the proton and neutron are $2.792\,\mu_N$ and $-1.913\,\mu_N$ respectively, where $\mu_N$ is the nuclear magneton defined as

$$\mu_N = \frac{e\hbar}{2m_\mathrm{p}}.$$

The origin of the proton and neutron anomalous magnetic moments can be explained in terms of the magnetic moments of the individual quarks and the baryon wavefunctions derived above.

Since quarks are fundamental Dirac fermions, the operators for the total magnetic moment and $z$-component of the magnetic moment are

$$\hat{\mu} = Q\frac{e}{m}\hat{\mathbf{S}} \quad \text{and} \quad \hat{\mu}_z = Q\frac{e}{m}\hat{S}_z.$$

For spin-up ($m_s = +\frac{1}{2}$) quarks, the expectation values of the $z$-component of the magnetic moment of the up- and down-quarks are

$$\mu_\mathrm{u} = \langle \mathrm{u}\uparrow|\hat{\mu}_z|\mathrm{u}\uparrow\rangle = \left(+\tfrac{2}{3}\right)\frac{e\hbar}{2m_\mathrm{u}} = +\frac{2m_\mathrm{p}}{3m_\mathrm{u}}\mu_N, \tag{9.42}$$

$$\mu_\mathrm{d} = \langle \mathrm{d}\uparrow|\hat{\mu}_z|\mathrm{d}\uparrow\rangle = \left(-\tfrac{1}{3}\right)\frac{e\hbar}{2m_\mathrm{d}} = -\frac{m_\mathrm{p}}{3m_\mathrm{d}}\mu_N. \tag{9.43}$$

The corresponding expressions for the spin-down states are

$$\langle \mathrm{u}\downarrow|\hat{\mu}_z|\mathrm{u}\downarrow\rangle = -\mu_\mathrm{u} \quad \text{and} \quad \langle \mathrm{d}\downarrow|\hat{\mu}_z|\mathrm{d}\downarrow\rangle = -\mu_\mathrm{d}.$$

The total magnetic moment of a baryon is the vector sum of the magnetic moments of the three constituent quarks

$$\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{\mu}}^{(1)} + \hat{\boldsymbol{\mu}}^{(2)} + \hat{\boldsymbol{\mu}}^{(3)},$$

where $\hat{\boldsymbol{\mu}}^{(i)}$ is the magnetic moment operator which acts on the $i$th quark. Therefore, the magnetic moment of the proton can be written

$$\mu_\mathrm{p} = \langle\hat{\mu}_z\rangle = \langle \mathrm{p}\uparrow|\hat{\mu}_z^{(1)} + \hat{\mu}_z^{(2)} + \hat{\mu}_z^{(3)}|\mathrm{p}\uparrow\rangle. \tag{9.44}$$

The order that the quarks appear in the proton wavefunction does not affect the calculation of the magnetic moment and it is sufficient to write

$$|\mathrm{p}\uparrow\rangle = \frac{1}{\sqrt{6}}(2\mathrm{u}\uparrow\mathrm{u}\uparrow\mathrm{d}\downarrow - \mathrm{u}\uparrow\mathrm{u}\downarrow\mathrm{d}\uparrow - \mathrm{u}\downarrow\mathrm{u}\uparrow\mathrm{d}\uparrow),$$

and thus (9.44) can be written as

$$\mu_\mathrm{p} = \tfrac{1}{6}\langle(2\mathrm{u}\uparrow\mathrm{u}\uparrow\mathrm{d}\downarrow - \mathrm{u}\uparrow\mathrm{u}\downarrow\mathrm{d}\uparrow - \mathrm{u}\downarrow\mathrm{u}\uparrow\mathrm{d}\uparrow)|\hat{\mu}_z|(2\mathrm{u}\uparrow\mathrm{u}\uparrow\mathrm{d}\downarrow - \mathrm{u}\uparrow\mathrm{u}\downarrow\mathrm{d}\uparrow - \mathrm{u}\downarrow\mathrm{u}\uparrow\mathrm{d}\uparrow)\rangle,$$

where $\hat{\mu}_z = \hat{\mu}_z^{(1)} + \hat{\mu}_z^{(2)} + \hat{\mu}_z^{(3)}$. Because of the orthogonality of the quark flavour and spin states, for example $\langle\mathrm{u}\uparrow\mathrm{u}\uparrow\mathrm{d}\downarrow\,|\mathrm{u}\downarrow\mathrm{u}\uparrow\mathrm{d}\uparrow\rangle = 0$, the expression for the proton magnetic moment reduces to

$$\begin{aligned}\mu_\mathrm{p} = {} & \tfrac{4}{6}\langle\mathrm{u}\uparrow\mathrm{u}\uparrow\mathrm{d}\downarrow|\hat{\mu}_z|\mathrm{u}\uparrow\mathrm{u}\uparrow\mathrm{d}\downarrow\rangle + \tfrac{1}{6}\langle\mathrm{u}\uparrow\mathrm{u}\downarrow\mathrm{d}\uparrow|\hat{\mu}_z|\mathrm{u}\uparrow\mathrm{u}\downarrow\mathrm{d}\uparrow\rangle \\ & + \tfrac{1}{6}\langle\mathrm{u}\downarrow\mathrm{u}\uparrow\mathrm{d}\uparrow|\hat{\mu}_z|\mathrm{u}\downarrow\mathrm{u}\uparrow\mathrm{d}\uparrow\rangle. \end{aligned} \tag{9.45}$$

Equation (9.45) can be evaluated using

$$\begin{aligned} \hat{\mu}_z|\mathrm{u}\uparrow\rangle &= +\mu_\mathrm{u}\,|\mathrm{u}\uparrow\rangle \quad \text{and} \quad \hat{\mu}_z|\mathrm{u}\downarrow\rangle = -\mu_\mathrm{u}\,|\mathrm{u}\downarrow\rangle, \\ \hat{\mu}_z|\mathrm{d}\uparrow\rangle &= +\mu_\mathrm{d}\,|\mathrm{d}\uparrow\rangle \quad \text{and} \quad \hat{\mu}_z|\mathrm{d}\downarrow\rangle = -\mu_\mathrm{d}\,|\mathrm{d}\downarrow\rangle, \end{aligned}$$

giving

$$\mu_\mathrm{p} = \tfrac{4}{6}(\mu_\mathrm{u} + \mu_\mathrm{u} - \mu_\mathrm{d}) + \tfrac{1}{6}(\mu_\mathrm{u} - \mu_\mathrm{u} + \mu_\mathrm{d}) + \tfrac{1}{6}(-\mu_\mathrm{u} + \mu_\mathrm{u} + \mu_\mathrm{d}).$$

Therefore, the quark model prediction for the magnetic moment of the proton is

$$\mu_\mathrm{p} = \tfrac{4}{3}\mu_\mathrm{u} - \tfrac{1}{3}\mu_\mathrm{d}.$$

The prediction for the magnetic moment of the neutron can be written down by replacing u → d and vice versa,

$$\mu_\mathrm{n} = \tfrac{4}{3}\mu_\mathrm{d} - \tfrac{1}{3}\mu_\mathrm{u}.$$

Assuming that $m_\mathrm{u} \approx m_\mathrm{d}$, the relations of (9.42) and (9.43) imply that $\mu_\mathrm{u} = -2\mu_\mathrm{d}$. Consequently, the ratio of the proton and neutron magnetic moments is predicted to be

$$\frac{\mu_\mathrm{p}}{\mu_\mathrm{n}} = \frac{4\mu_\mathrm{u} - \mu_\mathrm{d}}{4\mu_\mathrm{d} - \mu_\mathrm{u}} = -\frac{3}{2},$$

which is in reasonable agreement with the experimentally measured value of −1.46. The best agreement between the quark model predictions and the measured values of the magnetic moments of the $L = 0$ baryons is obtained with

$$m_\mathrm{u} = 0.338\,\mathrm{GeV}, \quad m_\mathrm{d} = 0.322\,\mathrm{GeV} \quad \text{and} \quad m_\mathrm{s} = 0.510\,\mathrm{GeV}.$$

Using these values in (9.42) and (9.43) gives $\mu_\mathrm{u} = +1.85\mu_N$ and $\mu_\mathrm{d} = -0.97\mu_N$, reproducing the observed values of the proton and neutron magnetic moments.