AKT II – Übung 4

9. 4. 2019

Bsp 1

4.4 For a particle with four-momentum $p^\mu = (E, \mathbf{p})$, the general solution to the free-particle Dirac Equation can be written

$$\psi(p) = [au_1(p) + bu_2(p)]e^{i(\mathbf{p}\cdot\mathbf{x}-Et)}.$$

Using the explicit forms for u_1 and u_2, show that the four-vector current $j^\mu = (\rho, \mathbf{j})$ is given by

$$j^\mu = 2p^\mu.$$

Furthermore, show that the resulting probability density and probability current are consistent with a particle moving with velocity $\beta = \mathrm{p}/E$.

Bsp 2

4.5 Writing the four-component spinor u_1 in terms of two two-component vectors

$$u = \begin{pmatrix} u_A \\ u_B \end{pmatrix},$$

show that in the non-relativistic limit, where $\beta \equiv v/c \ll 1$, the components of u_B are smaller than those of u_A by a factor v/c.

Bsp 3

4.12 Verify that the helicity operator

$$\hat{h} = \frac{\hat{\boldsymbol{\Sigma}} \cdot \hat{\mathbf{p}}}{2p} = \frac{1}{2p}\begin{pmatrix} \boldsymbol{\sigma} \cdot \hat{\mathbf{p}} & 0 \\ 0 & \boldsymbol{\sigma} \cdot \hat{\mathbf{p}} \end{pmatrix},$$

commutes with the Dirac Hamiltonian,

$$\hat{H}_D = \boldsymbol{\alpha} \cdot \hat{\mathbf{p}} + \beta m.$$

Bsp 4

4.14 Under the combined operation of parity and charge conjugation ($\hat{C}\hat{P}$) spinors transform as

$$\psi \rightarrow \psi^c = \hat{C}\hat{P}\psi = i\gamma^2\gamma^0\psi^*.$$

Show that up to an overall complex phase factor

$$\hat{C}\hat{P}u_\uparrow(\theta, \phi) = v_\downarrow(\pi - \theta, \pi + \phi).$$