AKT II – Übung

30. 4. 2019

Bsp 1

6.7 Using helicity amplitudes, calculate the differential cross section for $e^-\mu^- \to e^-\mu^-$ scattering in the following steps:

a) From the Feynman rules for QED, show that the lowest-order QED matrix element for $e^-\mu^- \to e^-\mu^-$ is

$$\mathcal{M}_{fi} = -\frac{e^2}{(p_1 - p_3)^2} g_{\mu\nu} \left[\overline{u}(p_3)\gamma^\mu u(p_1)\right] \left[\overline{u}(p_4)\gamma^\nu u(p_2)\right],$$

where p_1 and p_3 are the four-momenta of the initial- and final-state e^-, and p_2 and p_4 are the four-momenta of the initial- and final-state μ^-.

b) Working in the centre-of-mass frame, and writing the four-momenta of the initial- and final-state e^- as $p_1^\mu = (E_1, 0, 0, \mathrm{p})$ and $p_3^\mu = (E_1, p\sin\theta, 0, \mathrm{p}\cos\theta)$ respectively, show that the electron currents for the four possible helicity combinations are

$$\begin{aligned}
\overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1) &= 2(E_1 c, \mathrm{p}s, -i\mathrm{p}s, \mathrm{p}c),\\
\overline{u}_\uparrow(p_3)\gamma^\mu u_\downarrow(p_1) &= 2(ms, 0, 0, 0),\\
\overline{u}_\uparrow(p_3)\gamma^\mu u_\uparrow(p_1) &= 2(E_1 c, \mathrm{p}s, i\mathrm{p}s, \mathrm{p}c),\\
\overline{u}_\downarrow(p_3)\gamma^\mu u_\uparrow(p_1) &= -2(ms, 0, 0, 0),
\end{aligned}$$

where m is the electron mass, $s = \sin(\theta/2)$ and $c = \cos(\theta/2)$.

c) Explain why the effect of the parity operator $\hat{P} = \gamma^0$ is

$$\hat{P}u_\uparrow(\mathrm{p}, \theta, \phi) = \hat{P}u_\downarrow(\mathrm{p}, \pi - \theta, \pi + \phi)\,.$$

Hence, or otherwise, show that the muon currents for the four helicity combinations are

$$\begin{aligned}
\overline{u}_\downarrow(p_4)\gamma^\mu u_\downarrow(p_2) &= 2(E_2 c, -\mathrm{p}s, -i\mathrm{p}s, -\mathrm{p}c)\,,\\
\overline{u}_\uparrow(p_4)\gamma^\mu u_\downarrow(p_2) &= 2(Ms, 0, 0, 0)\,,\\
\overline{u}_\uparrow(p_4)\gamma^\mu u_\uparrow(p_2) &= 2(E_2 c, -\mathrm{p}s, i\mathrm{p}s, -\mathrm{p}c)\,,\\
\overline{u}_\downarrow(p_4)\gamma^\mu u_\uparrow(p_2) &= -2(Ms, 0, 0, 0)\,,
\end{aligned}$$

where M is the muon mass.

d) For the relativistic limit where $E \gg M$, show that the matrix element squared for the case where the incoming e^- and incoming μ^- are both left-handed is given by

$$|\mathcal{M}_{LL}|^2 = \frac{4e^4 s^2}{(p_1 - p_3)^4}\,,$$

where $s = (p_1 + p_2)^2$. Find the corresponding expressions for $|\mathcal{M}_{RL}|^2$, $|\mathcal{M}_{RR}|^2$ and $|\mathcal{M}_{LR}|^2$.

e) In this relativistic limit, show that the differential cross section for unpolarised $e^-\mu^- \to e^-\mu^-$ scattering in the centre-of-mass frame is

$$\frac{d\sigma}{d\Omega} = \frac{2\alpha^2}{s} \cdot \frac{1 + \frac{1}{4}(1 + \cos\theta)^2}{(1 - \cos\theta)^2}\,.$$

Bsp 2

7.4 For a spherically symmetric charge distribution $\rho(\mathrm{r})$, where

$$\int \rho(\mathrm{r})\,\mathrm{d}^3\mathbf{r} = 1\,,$$

show that the form factor can be expressed as

$$\begin{aligned} F(\mathbf{q}^2) &= \frac{4\pi}{\mathrm{q}} \int_0^\infty \mathrm{r}\sin(\mathrm{qr})\rho(\mathrm{r})\,\mathrm{dr}\,, \\ &\simeq 1 - \frac{1}{6}\mathrm{q}^2\langle \mathrm{R}^2\rangle + \ldots\,, \end{aligned}$$

where $\langle \mathrm{R}^2\rangle$ is the mean square charge radius. Hence show that

$$\langle \mathrm{R}^2\rangle = -6\left[\frac{\mathrm{d}F(\mathbf{q}^2)}{\mathrm{dq}^2}\right]_{\mathrm{q}^2=0}\,.$$