6.0/4.0 VU Formale Methoden der Informatik 185.291 SS 2011 28 October 2011				
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1.) Consider the following problem:

> **IMPERFECT-COMPRESSION(IC)**
>
> INSTANCE: A program (i.e. a source code) Π such that Π takes one string as input and outputs a string. It is guaranteed that Π terminates on any input string.
>
> QUESTION: Does there exists an input string I for Π such that $|\Pi(I)| > |I|$. Here $|J|$ denotes the length of a string J, and $\Pi(J)$ is the string returned by Π on input string J.

Prove that the problem **IC** is semi-decidable. For this, describe a procedure that shows the semi-decidability of the problem (i.e. a semi-decision procedure for **IC**) and argue that it is correct. **(15 points)**

2.) Let φ be the formula $(x \to (y \to z)) \to ((x \wedge y) \to z)$.

(a) Use the Tseitin translation and compute $\hat{\delta}(\varphi)$. (Hint: It is allowed to avoid the labels for atoms; use the atoms instead. Moreover, use ℓ_φ as the label for φ.) **(5 points)**

(b) Try to derive the empty clause $\Box$ from

$$\left(\bigwedge_{D \in \hat{\delta}(\varphi)} D \right) \wedge \neg \ell_\varphi$$

by resolution. **(4 points)**

(c) Answer the following questions and explain in detail.

i. Is $\bigwedge_{D \in \hat{\delta}(\varphi)} D$ satisfiable? If so then provide a model.

ii. Is $\bigwedge_{D \in \hat{\delta}(\varphi)} D$ valid? If not then provide a counterexample.

iii. Is $\left(\bigwedge_{D \in \hat{\delta}(\varphi)} D \right) \to \ell_\varphi$ valid?

iv. Is φ valid?

(6 points)

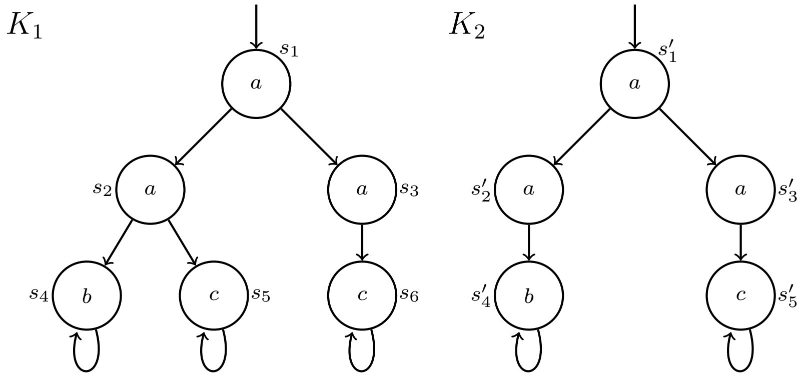
3.) Show that the following correctness assertion is totally correct. Describe the function computed by the program if we consider a as its input and c as its output.

$\{ \mathbf{1\colon} \boldsymbol{a \geq 0} \}$
$b \leftarrow 1;$
$c \leftarrow 0;$
$\{ \boldsymbol{Inv\colon b = (c+1)^3 \wedge 0 \leq c^3 \leq a} \}$
while $b \leq a$ do
  $d \leftarrow 3 * c + 6;$
  $c \leftarrow c + 1;$
  $b \leftarrow b + c * d + 1$
od
$\{ \mathbf{2\colon} \boldsymbol{c^3 \leq a < (c+1)^3} \}$

(15 points)

4.) Simulation and Bisimulation

(a) Let K_1 and K_2 be the two Kripke structures given below. Check which of the relations $K_1 \leq K_2$, $K_1 \geq K_2$, $K_1 \equiv K_2$ hold on K_1 and K_2. Justify your answer.



(5 points)

(b) Show that simulation is a transitive relation: Given any 3 Kripke structures $K_1 = \{S_1, R_1, L_1\}$, $K_2 = \{S_2, R_2, L_2\}$ and $K_3 = \{S_3, R_3, L_3\}$ such that $K_1 \leq K_2$ and $K_2 \leq K_3$, it holds that $K_1 \leq K_3$. **(5 points)**

(c) Given the Kripke structures $K_1 = (S_1, R_1, L_1)$, $K_2 = (S_2, R_2, L_2)$, $K_3 = (S_3, R_3, L_3)$, the simulation relation $H_1 \subseteq S_1 \times S_2$ from K_1 to K_2 and the simulation relation $H_2 \subseteq S_2 \times S_3$ from K_2 to K_3, state an algorithm which computes a simulation H_3 from K_1 to K_3. **(5 points)**