6.0/4.0 VU Formale Methoden der Informatik

185.291 **SS 2012** **19 October 2012**

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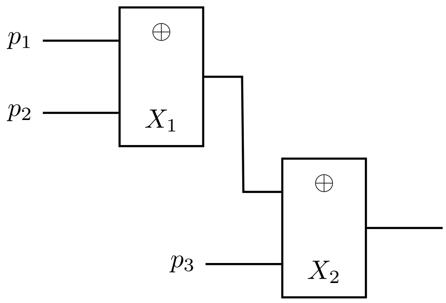
1.) Consider the following problem:

> **PAIRS**
>
> INSTANCE: A program Π such that Π takes as input a pair of strings and outputs *true* or *false*. It is guaranteed that Π terminates on any input.
>
> QUESTION: Does there exist a pair (I_1, I_2) of strings such that Π terminates on (I_1, I_2) with output value *true*? That is, does there exist I_1, I_2 such that $\Pi(I_1, I_2) = \mathit{true}$?

Prove that the problem **PAIRS** is semi-decidable. For this, describe a procedure that shows the semi-decidability of the problem (i.e. a semi-decision procedure for **PAIRS**) and argue that it is correct.

(15 points)

2.) (a) Given the following circuit:



- Apply Tseitin's transformation to it, to obtain a set $\mathcal{D}$ of clauses that encodes the same function as the circuit.
- Describe in your own words (not as a formula) what the circuit computes.

Hint: For the translation of XOR ($\oplus$), you may use that
$(a \oplus b) \equiv (a \vee b) \wedge (\neg a \vee \neg b)$. **(6 points)**

(b) Consider a simplified variant of Tseitin's transformation: let φ be a propositional formula, let $\Sigma(\varphi)$ be the set of all subformulas of φ, and let ℓ_φ be the label for φ. Then, the result of simplified Tseitin's transformation is the formula:

$$\lambda = \left(\bigwedge_{\psi \in \Sigma(\varphi)} (\ell_\psi \leftrightarrow \psi) \right) \rightarrow \ell_\varphi$$

Prove: λ is valid if and only if φ is valid.

(9 points)

3.) (a) Show that the following version of the 'logical consequence'-rule is not sound.

$$\frac{F \Rightarrow F' \quad \{\, F \,\}\, p \,\{\, G \,\}}{\{\, F' \,\}\, p \,\{\, G \,\}}$$

In words, the rule states: If $F \Rightarrow F'$ is a valid formula and if the correctness assertion $\{\, F \,\}\, p \,\{\, G \,\}$ is true regarding partial/total correctness, then the assertion $\{\, F' \,\}\, p \,\{\, G \,\}$ is also true regarding partial/total correctness. Show that this is not necessarily the case, by giving a counter-example; argue why it is a counter-example. **(5 points)**

(b) Show that the following correctness assertion is totally correct.

Hint: Depending on how you choose the variant, use one of the following annotation rules:

$\mathsf{while}\ e\ \mathsf{do} \cdots \mathsf{od} \mapsto \{\ Inv\ \}\mathsf{while}\ e\ \mathsf{do}\ \{\ Inv \land e \land t{=}t_0\ \} \cdots \{\ Inv \land 0{\le}t{<}t_0\ \}\mathsf{od}\{\ Inv \land \neg e\ \}$

$\mathsf{while}\ e\ \mathsf{do} \cdots \mathsf{od} \mapsto \{\ Inv\ \}\mathsf{while}\ e\ \mathsf{do}\ \{\ Inv \land e \land t{=}t_0\ \} \cdots \{\ Inv \land (e \Rightarrow 0{\le}t{<}t_0)\ \}\mathsf{od}\{\ Inv \land \neg e\ \}$

$\boldsymbol{\{\, n \geq 0 \,\}}$

$i \leftarrow 0;$

$s \leftarrow 0;$

$\boldsymbol{\{\, Inv\colon s = i(i-1) \land 0 \leq i \leq n+1 \,\}}$

$\mathsf{while}\ i \leq n\ \mathsf{do}$

$\quad s \leftarrow s + 2i;$

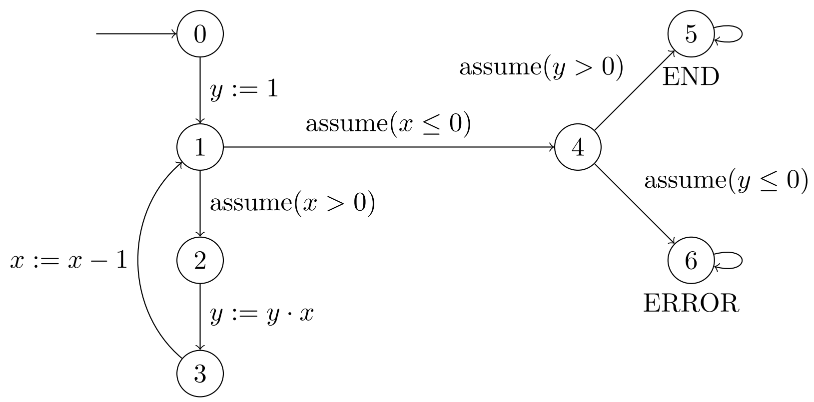
$\quad i \leftarrow i + 1$

$\mathsf{od};$

$\boldsymbol{\{\, s = n^2 + n \,\}}$

(10 points)

4.) Consider the following labeled transition system (LTS):



(a) Provide an abstraction for the LTS that uses the predicates $x > 0$ and $y > 0$. Please use the abbreviations p for $x > 0$, $\bar{p}$ for $x \leq 0$, q for $y > 0$, $\bar{q}$ for $y \leq 0$. **(5 points)**

(b) Give an ACTL formula that corresponds to the unreachability of the error location. **(2 points)**

(c) Assume that the variables x and y are 8-bit integers, i.e., the variables take values in the interval $[-128, 127]$. We model the labeled transition system as Kripke structure $M = (S, I, R, L)$, where

- the set of atomic propositions is $AP = \{ERROR\}$,
- $S = \{(c, x, y) \mid c \in [0, 6], x \in [-128, 127], y \in [-128, 127]\}$,
- $I = \{(0, x, y) \mid x \in [-128, 127], y \in [-128, 127]\}$,
- $R = \{((c, x, y), (c', x', y')) \mid$ there is a transition in the LTS from c to c' such that x, y go to x', y' $\}$,

  and
- $L(c, x, y) = \begin{cases} ERROR & \text{if } c = 6, \\ \neg ERROR & \text{otherwise.} \end{cases}$

Show that the abstraction (a) simulates the Kripke structure M. **(8 points)**