6.0/4.0 VU Formale Methoden der Informatik

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1.) Consider the following decision problem:

FREQUENCY ASSIGNMENT
INSTANCE: a triple (T, S, m), such that

- $T = t_1, t_2, ..., t_n$ is a set of transmitters
- $S \subseteq T^2$ is a set of pairs of transmitters that interfere and therefore cannot use the same frequency
- m is a positive integer that indicates the number of available frequencies.

QUESTION: Does there exist an assignment of each transmitter to one of m frequencies such that there is no interference between the transmitters? I.e., does there exist $f : T \to \{1, \ldots, m\}$, such that for every $i, j \in \{1, \ldots, n\}$, if $(t_i, t_j) \in S$, then $f(t_i) \neq f(t_j)$?

We want to prove the *NP-hardness* of **FREQUENCY ASSIGNMENT**. Your task is to give a polynomial time reduction R from **3-COLORABILITY** to **FREQUENCY ASSIGNMENT**. Additionally, prove the "$\Leftarrow$" direction in the proof of correctness of the reduction, i.e., let x denote an arbitrary instance of the **3-COLORABILITY** problem and let $R(x)$ denote the corresponding instance of the **FREQUENCY ASSIGNMENT** problem. You have to prove the following statement: if $R(x)$ is a positive instance of **FREQUENCY ASSIGNMENT**, then x is a positive instance of **3-COLORABILITY**.

(15 points)

2.) (a) Use Ackermann's reduction and translate

$$B(B(x)) \doteq B(A(x)) \to A(B(A(x))) \doteq y \vee C(x, y) \doteq C(B(x), A(x))$$

to a validity-equivalent E-formula φ^E. A, B, and C are function symbols, x and y are variables. **(3 points)**

(b) Let φ be any first-order formula. Prove that φ is satisfiable if and only if $\neg\varphi$ is *not* valid. **(3 points)**

(c) We define the set L of all lists as follows:

$$L ::= nil \mid (c : L)$$

nil denotes the empty list containing no element. Then $(1 : (2 : (3 : nil)))$ is the representation of the list $(1, 2, 3)$. We define the function *append* by $append(nil, y) = y$ and $append((c : x), y) = (c : append(x, y))$. Show that, for all lists ℓ, $append(\ell, nil) = \ell$ holds.
Hint: Use structural induction or apply induction on the length of lists. Please be mathematically precise. **(9 points)**

3.) Prove that the following correctness assertion is true regarding total correctness. Use the invariant $2x = y + 5z \wedge x \geq y$.

Some annotation rules that you might not remember:
abort $\mapsto$ { false } abort { false } $\qquad$ $\{F\}v := e \mapsto \{F\}v := e\{\exists v'(F[v/v'] \wedge v = e[v/v'])\}$
if e then $\{F\} \cdots$ else $\{G\} \mapsto \{(e \Rightarrow F) \wedge (\neg e \Rightarrow G)\}$if e then $\{F\} \cdots$ else $\{G\}$
$\{F\}$if e then $\cdots$ else $\mapsto \{F\}$if e then $\{F \wedge e\} \cdots$ else $\{G \wedge \neg e\}$
while e do $\cdots$ od $\mapsto \{\, Inv \,\}$while e do $\{\, Inv \wedge e \wedge t = t_0 \,\} \cdots \{\, Inv \wedge 0 \leq t < t_0 \,\}od\{\, Inv \wedge \neg e \,\}$
while e do $\cdots$ od $\mapsto \{\, Inv \,\}$while e do $\{\, Inv \wedge e \wedge t = t_0 \,\} \cdots \{\, Inv \wedge (e \Rightarrow 0 \leq t < t_0) \,\}od\{\, Inv \wedge \neg e \,\}$

$\{\, Pre\colon x = 2z \wedge y = z \wedge z > 0 \,\}$
$x := x + y$;
if $x > 0$ then
    while $x \neq y$ do
        $x := x + 1$;
        $y := y + 2$
    od
else
    abort
fi
$\{\, Post\colon x = y \,\}$

(15 points)

4.) Computation Tree Logic.

Let AP be a set of propositional symbols, and $AP' \subseteq AP$ be a subset of AP.

We recall the definition of ACTL formulae over AP:

- $p \in AP$ and $\neg p \in AP$ are ACTL formulae,
- if φ and ψ are ACTL formulae, then $\varphi \wedge \psi$, $\varphi \vee \psi$, $\mathbf{AX}\,\varphi$, $\mathbf{AG}\,\varphi$, and $\mathbf{A}\,[\varphi\,\mathbf{U}\,\psi]$ are ACTL formulae.

Let $M = (S, I, R, L)$ and $M' = (S', I', R', L')$ be two Kripke structures related as follows:

- $S = S'$, $I = I'$, $R = R'$, and
- $L'(s) = L(s) \cap AP'$, where $s \in S$.

Let $\hat{M} = (\hat{S}, \hat{I}, \hat{R}, \hat{L})$ be a Kripke structure related to M' as follows:

- $\hat{S} = 2^{AP'}$, i.e., a state $\hat{s} \in \hat{S}$ is a subset of AP',
- $\hat{I} = \{\hat{s} \in \hat{S} \mid \exists s \in I'.\ L'(s) = \hat{s}\}$, i.e., a state $\hat{s} \in \hat{S}$ is an initial state of $\hat{M}$ if there is an initial state $s \in I'$ such that s is labeled with $\hat{s}$.
- $\hat{R} = \{(\hat{s}, \hat{t}) \in \hat{S} \times \hat{S} \mid \exists s, t \in S.\ \hat{s} = L'(s) \wedge \hat{t} = L'(t) \wedge (s, t) \in R'\}$, i.e., for each transition $(\hat{s}, \hat{t}) \in \hat{R}$ there are states $s, t \in S'$ such that there is a transition from s to t and s is labeled with $\hat{s}$ and t is labeled with $\hat{t}$,
- $\hat{L}(\hat{s}) = \hat{s}$ for all $\hat{s} \in \hat{S}$, i.e., each state $\hat{s} \in \hat{S}$ is labeled with the atomic propositions it contains.

(a) Prove that for any *ACTL* formula φ over propositions from AP' the following holds:

$$M \models \varphi \text{ if and only if } M' \models \varphi$$

Hint: Use the semantics of ACTL. You can either use an induction on the structure of the formula (structural induction) or an induction on the formula length.

(5 points)

(b) Prove that for any ACTL formula φ over propositions from AP' the following holds:

$$\text{If } \hat{M} \models \varphi, \text{ then } M' \models \varphi$$

Hint: You can use the following theorem from the lecture:
Let M_1 and M_2 be Kripke structures such that $M_1 \preceq M_2$. Let φ be an ACTL formula. If $M_2 \models \varphi$, then $M_1 \models \varphi$.*

(10 points)