

# VU Logic and Computation

## Exercises – Computability Theory

(Fixed) Deadline December 28th, 2012

December 13, 2012

*Notation:* We use  $\Phi_k$  to denote the  $k$ -th computable function and  $Dom(\Phi_k)$  for the set  $\{x \mid \Phi_k(x) \downarrow\}$ .

1. Exhibit a lambda term which simulates the boolean function "not" (hint: encode true **T** by  $\lambda xy.x$  and false **F** by  $\lambda xy.y$ )
2. Is the set  $\{i \mid Dom(\Phi_i) = \emptyset\}$  recursive, recursive enumerable or none of them?  
(A formal proof is needed)
3. Is there an index  $p$  such that  $\Phi_p$  is the constant function always equal to  $p + 2$ ?  
(A formal proof is needed)
4. Show that the function

$$f_1(x) = \begin{cases} \min n \text{ such that } \Phi_x(n) \downarrow & \text{if } \exists n \Phi_x(n) \downarrow \\ 0 & \text{otherwise} \end{cases}$$

is *not* computable.  
(A formal proof is needed)

5. (\*) Show that the function

$$f_2(x) = \begin{cases} \min n \text{ such that } \Phi_x(n) \downarrow & \text{if } \exists n \Phi_x(n) \downarrow \\ \uparrow & \text{otherwise} \end{cases}$$

is *not* computable

*Note:* The exercise marked by (\*) is optional (= not compulsory)