VU Logic and Computation
Exercises: *Mathematical Logic*

October 29, 2012

1. Formalize the following sentences in first-order classical logic:
   (a) Two orthogonal lines have a common point (that is, a point that belongs to both lines)
   (b) If two lines are parallel, then they do not have a common point
   (c) through each point outside a line there passes a parallel to this line
2. Formalize in first-order classical logic the data structure Queue
3. Exhibit derivations in natural deduction for the formulas
   - $(A \to B) \vee (B \to A)$
   - $(\neg A \to A) \to A$
4. For each of the following formulas provide a sequent calculus proof (if the formula is valid) or a countermodel (if the formula is not valid)
   (a) $\forall x((\exists y C(x, y)) \to A(x)) \to \forall x \exists y(C(x, y) \to A(x))$
   (b) $\exists x(P(x) \to \forall y P(y))$
   (c) $(\forall x A(x) \to \exists y B(x, y)) \to \exists x(A(x) \to \exists y B(x, y))$
5. Let P be the formula $\exists x A(x) \wedge \exists x \neg A(x)$. Is P satisfiable? In the affirmative case, how many elements does the domain of each model have?
6. Let $\mathcal{L}$ be the first-order language with two binary predicate symbols: equality and $E(x, y)$. Every structure $\mathcal{A}$ can be interpreted as a directed graph G where the elements in $D_{\mathcal{A}}$ are the nodes of G and there exists an edge from a to b if and only if $v^{\mathcal{A}}(E(a, b)) = 1$. Similarly every directed graph can be interpreted as a structure for $\mathcal{L}$.
   - Show that there is no first-order formula on $\mathcal{L}$ which is true if and only if a given directed graph G is infinite. Can this property be expressed by an *infinite* set of first-order formulas?
   - Can an infinite set of first-order formulas express that a given directed graph G is *finite*?

Submit your solutions to agata@logic.at till

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(an acknowledgment will be sent after the receipt of your email)