

VU Logic and Computation Exercises – Computability Theory

December 19, 2013

Preliminary information

- I expect *individual* solutions (no group work)
- The exercises should be written in Latex (preferred) or Word
- The (fix!) deadline for the submission is **January 10th** 2014 (9:00am)
- The exercises should be sent via email (.pdf format) to

agata@logic.at

(an acknowledgment will be sent after the receipt of your email)

Exercises

Notation: We use Φ_k to denote the k -th computable function and $Dom(\Phi_k)$ for the set $\{x \mid \Phi_k(x) \downarrow\}$.

1. Are the following sets
 - (a) $\{i \mid \exists n, \Phi_i(n) \downarrow \text{ and } \Phi_i(n+1) \downarrow\}$
 - (b) $\{i \mid Dom(\Phi_i) \cap Dom(\Phi_a) = \emptyset\}$ in case $Dom(\Phi_a) \neq \emptyset$
 - (c) $\{i \mid Dom(\Phi_i) \cap Dom(\Phi_a) = \emptyset\}$ in case $Dom(\Phi_a) = \emptyset$recursive, r.e. or none of them? (Prove your claims)
2. Prove that there is an index p such that $\Phi_p(0) = p^2$?
3. Exhibit a lambda term which simulates the boolean function " \leftrightarrow " (i.e. $A \leftrightarrow B$ is true if and only if either $A = B = \mathbf{T}$ or $A = B = \mathbf{F}$)
(hint: encode true \mathbf{T} by $\lambda xy.x$ and false \mathbf{F} by $\lambda xy.y$)