VU Logic and Computation
Exercises: *Classical Logic*

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Preliminary information

- I expect *individual* solutions (no group work)
- The exercises should be written in Latex (preferred) or Word
- The (fix!) deadline for the submission is **November 26** 2013 (12:00)
- The exercises should be given to me (hard copy, after the class on November 26th) *and* sent via email (.pdf format) to

  agata@logic.at

- In case you cannot bring the exercises after the class, you can send them via email by the same date/time (an acknowledgment will be sent after the receipt of your email)

Exercises

1. (a) Formalize the following sentence in classical logic

   There is a person that if he votes for X then everybody votes for X

   (b) exhibit a *natural deduction* derivation for the resulting formula (in case it is valid) or an interpretation that falsifies it (in case it is not valid).

2. For each of the following statements provide either a proof[1] (if the statement holds) or a counterexample (if the statement does not hold)

   - $\forall x P(x) \to \forall x Q(x) \models \forall x(P(x) \to Q(x))$
   - $\forall x(P(x) \leftrightarrow Q(x))) \models \exists x P(x) \leftrightarrow \exists x Q(x)$
     (where the notation $X \leftrightarrow Y$ abbreviates $(X \to Y) \wedge (Y \to X)$).

3. Consider the formula

   $$\forall x \exists y A(x,y) \wedge \forall x \neg A(x,x) \wedge \forall xyz[(A(x,y) \wedge A(y,z)) \to A(x,z)]$$

   - Is the formula satisfiable, unsatisfiable or valid?
   - Is the formula satisfiable in interpretations with finite domains?

4. Let us denote by pLJ^+ the sequent calculus pLJ (see the Appendix) *extended with* the axiom (schema) $\vdash A \vee \neg A$. (a) Is the sequent $\vdash \neg\neg A \to A$ derivable in pLJ^+? (in the affirmative case show the derivation). (b) Is $\vdash \neg\neg A \to A$ derivable in pLJ^+ without using the (cut) rule? (Explain)

5. Exhibit a formula that is satisfiable in *all and only* the interpretations having k elements ($k > 2$) in their domains.

[1] Using the proof theoretic *or* the model theoretic approach.

Appendix: Sequent Calculus pLJ

Axioms and cut:

$$A \vdash A \qquad \frac{\Gamma \vdash A \qquad \Sigma, A \vdash B}{\Gamma, \Sigma \vdash B}\ (\text{CUT})$$

Structural Rules:

$$\frac{\Gamma, B, A, \Delta \vdash C}{\Gamma, A, B, \Delta \vdash C}\ (\text{exchange,l}) \qquad \frac{\Gamma, A, A, \Delta \vdash C}{\Gamma, A, \Delta \vdash C}\ (\text{contraction,l})$$

$$\frac{\Gamma \vdash B}{\Gamma, A \vdash B}\ (\text{weakening,l}) \qquad \frac{\Gamma \vdash}{\Gamma \vdash B}\ (\text{weakening,r})$$

Logical Rules:

$$\frac{\Gamma, A \vdash C \qquad \Gamma, B \vdash C}{\Gamma, A \vee B \vdash C}\ (\vee, l) \qquad \frac{\Gamma \vdash A_i}{\Gamma \vdash A_1 \vee A_2}\ (\vee, r)_i$$

$$\frac{\Gamma, A_i \vdash C}{\Gamma, A_1 \wedge A_2 \vdash C}\ (\wedge, l)_i \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \wedge B}\ (\wedge, r)$$

$$\frac{\Gamma, B \vdash C \qquad \Gamma \vdash A}{\Gamma, A \to B \vdash C}\ (\to, l) \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \to B}\ (\to, r)$$

$$\frac{\Gamma \vdash A}{\Gamma, \neg A \vdash}\ (\neg, l) \qquad \frac{\Gamma, A \vdash}{\Gamma \vdash \neg A}\ (\neg, r)$$