VU Logic and Computation
Exercises – Computability Theory

December 19, 2014

Preliminary information

- I expect *individual* solutions (no group work)

- The exercises should be written in Latex (preferred) or Word

- The (fix!) deadlines for the submission are:

  - **January 9th** 2015 12:00 *am*, if you want individual feedback on January 15th.

  - **January 14th** 2015 23:59, if you do not want individual feedback on January 15th.

  No deadline extensions possible.

- The exercises should be sent via email (.pdf format) to

  agata@logic.at

  and the file should be named

  ∗Computability.pdf

  (where∗ is to be instantiated with your surname)

- An acknowledgement of receipt will be sent after the submission (no acknowledgemnt means no receipt)

- Suggestion: do not try to solve the exercises short before the deadline (and do not ask for any explanation on the exercises after January 7th, 2015 (12:00 am))

1

Exercises

Notation: We use Φ_k to denote the k-th computable function.

1. Lambda calculus

   - Exhibit a lambda term which encodes the function $f : \mathcal{N} \to \mathcal{N}$ s.t.

   $$f(x) = 1 + \ldots (x - 1) + x$$

   - In arithmetic non every function has a fixpoint. Why, instead, every lambda term has a fixpoint?

2. Prove or disprove the following statements:

   - every countable union of recursive sets is recursive
   - every countable union of r.e. sets is r.e.

3. Is there is an index $p \in \mathcal{N}$ such that $\Phi_p \simeq \Phi_{p+1}$? Motivate your answer

4. Is the following function

   $$g(x, y) = \begin{cases} 1 & \text{if } \forall z \ (\Phi_x(z) \downarrow \wedge \Phi_y(z) \downarrow \Rightarrow \ \Phi_x(z) = \Phi_y(z)) \\ \uparrow & \text{otherwise} \end{cases}$$

   computable? Motivate your answer