VU Logic and Computation
Exercises: *Classical Logic*

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November 4, 2014

Preliminary information

- I expect *individual* solutions (no group work)
- The exercises should be written in Latex (preferred) or Word
- The (fixed!) deadline for the submission is Friday **November 28** 2014 (16:00)
- The exercises should be sent via email (.pdf format) to

  agata@logic.at

- an acknowledgement of receipt will be sent after the submission (no acknowledgemnt means no receipt)
- Suggestion: do not try to solve the exercises short before the deadline (and do not ask for any explanation on the exercises November 27th or November 28th)

Exercises

The exercise $(*)$ is optional.

1. For each of the following statements provide either a proof[1] (if the statement holds in classical logic) or a formal counterexample (if the statement does not hold in classical logic):

   1.1 $\models \exists x(\exists y A(y) \to A(x))$

   1.2 $\exists x(B(x) \to \forall y B(y)) \models \neg\neg\exists x B(x)$

2. Consider the sequent calculus obtained by adding the axiom

$$\vdash (\neg A \to A) \to A$$

   to the sequent calculus LJ (including (CUT)) for propositional intuitionistic logic.

   2.1 Exhibit a formula that can be derived in this calculus and that cannot be derived without using the (CUT) rule.

   2.2 $(*)$ Is LJ extended with $\vdash (\neg A \to A) \to A$ a calculus for propositional classical logic? Motivate your answer.

3. A graph is 5-colorable if there is a way of coloring its vertices with one of the 5 colours such that no two adjacent vertices share the same color.

   3.1 Let G be a graph. Define a set Σ of formulas which has a model if and only if G is 5-colorable.

   3.2 Let G' be an infinite graph. Knowing that every finite subgraph $G_0 \subset G'$ is 5-colorable can we conclude that G' is 5-colorable? Motivate your answer.

   (Hint for the formalization: you can use propositional logic and consider atoms: $color_{i,j}$ and $edge_{k,l}$ for each vertex v_i, v_k, v_l and color j)

[1] Using the proof theoretic *or* the model theoretic approach.

Appendix: Sequent Calculus LJ

Axioms and cut:

$$A \vdash A \qquad \frac{\Gamma \vdash A \qquad \Sigma, A \vdash B}{\Gamma, \Sigma \vdash B}\ (\text{CUT})$$

Structural Rules:

$$\frac{\Gamma, B, A, \Delta \vdash C}{\Gamma, A, B, \Delta \vdash C}\ (\text{exchange,l}) \qquad \frac{\Gamma, A, A, \Delta \vdash C}{\Gamma, A, \Delta \vdash C}\ (\text{contraction,l})$$

$$\frac{\Gamma \vdash B}{\Gamma, A \vdash B}\ (\text{weakening,l}) \qquad \frac{\Gamma \vdash}{\Gamma \vdash B}\ (\text{weakening,r})$$

Logical Rules:

$$\frac{\Gamma, A \vdash C \qquad \Gamma, B \vdash C}{\Gamma, A \vee B \vdash C}\ (\vee, l) \qquad \frac{\Gamma \vdash A_i}{\Gamma \vdash A_1 \vee A_2}\ (\vee, r)_{i \in \{1,2\}}$$

$$\frac{\Gamma, A_i \vdash C}{\Gamma, A_1 \wedge A_2 \vdash C}\ (\wedge, l)_{i \in \{1,2\}} \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \wedge B}\ (\wedge, r)$$

$$\frac{\Gamma, B \vdash C \qquad \Gamma \vdash A}{\Gamma, A \to B \vdash C}\ (\to, l) \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \to B}\ (\to, r)$$

$$\frac{\Gamma \vdash A}{\Gamma, \neg A \vdash}\ (\neg, l) \qquad \frac{\Gamma, A \vdash}{\Gamma \vdash \neg A}\ (\neg, r)$$