VORLESUNG

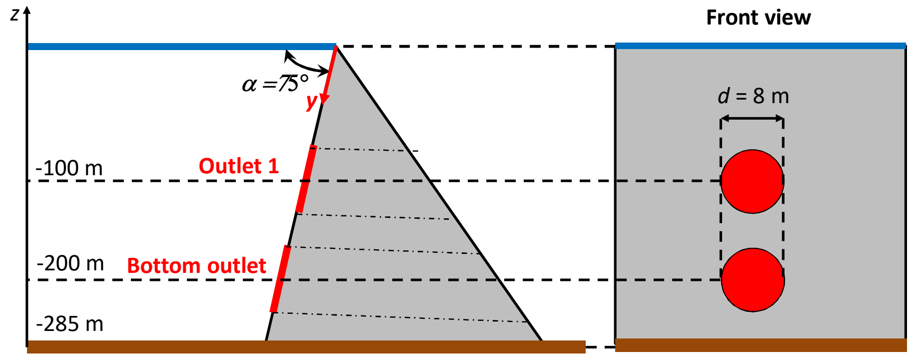
TECHNISCHE HYDRAULIK

222.564

Exercises

Hydrostatics – Laminar Flow – Turbulent Flow – Pipe Flow

Ex 1. Hydrostatics. Forces on submerged inclined planes



$$I_{\xi\xi} = \frac{1}{64}\pi d^4$$

What are the magnitude, direction and application point of the forces acting on both outlet gates ?

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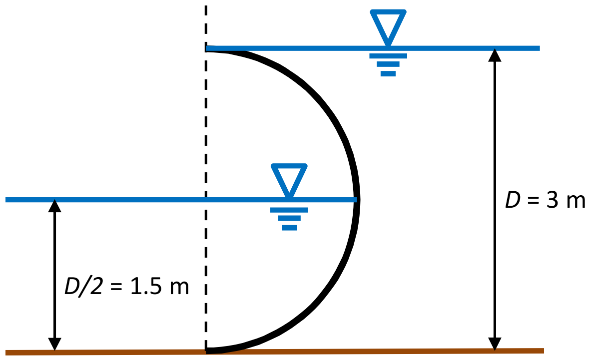
Exercise from the lecture TH_Hydrostatics

Ex 2. Hydrostatics. Forces on submerged curved planes



1) Draw the pressure distribution and force components on the structure
2) Compute the horizontal force component, the vertical force component, as well as the resultant force, its direction and its action point.

Note that all considerations are per unit width

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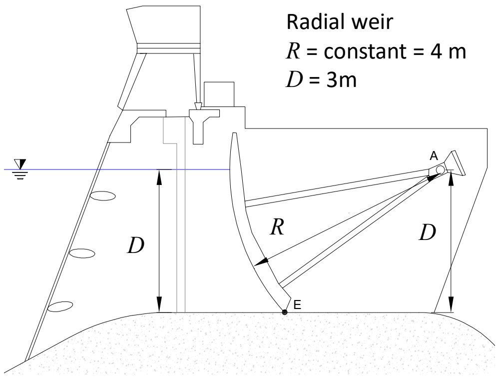
Exercise from the lecture TH_Hydrostatics

Ex 3. Hydrostatics. Forces on submerged curved planes



Radial weir
R = constant = 4 m
D = 3m

1) Draw the pressure distribution and force components on the structure
2) Compute the horizontal force component, the vertical force component, as well as the resultant force, its direction and its action point.

Note that all considerations are per unit width

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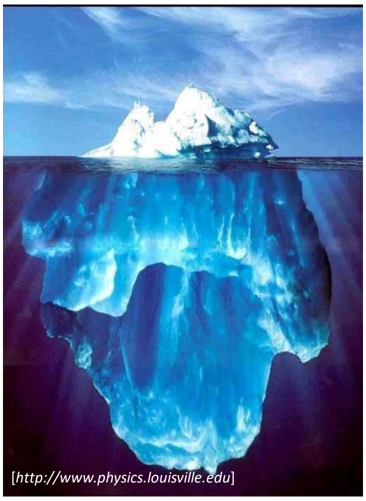
Exercise from the lecture TH_Hydrostatics

Ex 4. Hydrostatics. Archimedes principle – buoyant force



ρ_{water} (0°) = 999.8 kg m^{-3}
ρ_{ice} (0°) = 916.7 kg m^{-3}

What percentage volume of the iceberg sticks out above the water surfce ?

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Exercise from the lecture TH_Hydrostatics

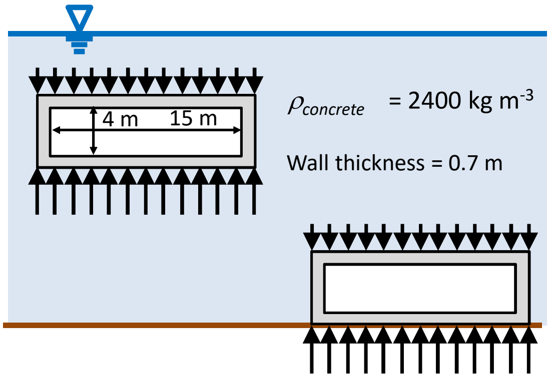
Ex 5. Hydrostatics. Archimedes principle – buoyant force

1) Will the tunnel element float or sink ?

2) If it floats, what is its stable position (by how much does it stick out of the water) ?

3) If it floats, what force is required to bring it down to the bottom ?

4) What is the resulting vertical force per unit length when the tunnel element lays on the bottom.



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Exercise from the lecture TH_Hydrostatics

Ex 6. Hydrostatics. Archimedes principle – buoyant force

Ein Behälter mit Wasser steht auf einer Waage. Man taucht den Finger ein – was passiert mit der Anzeige auf der Waage?

a) steigt
b) sinkt
c) bleibt gleich?

Begründung ?

Exercise from the lecture TH_Hydrostatics

Ex 7. Hydrostatics. Archimedes principle – buoyant force

Ein Fischer sitzt in einem Boot in einem kleinen Teich und wirft den Anker aus. Was passiert mit dem Wasserspiegel?

a) steigt

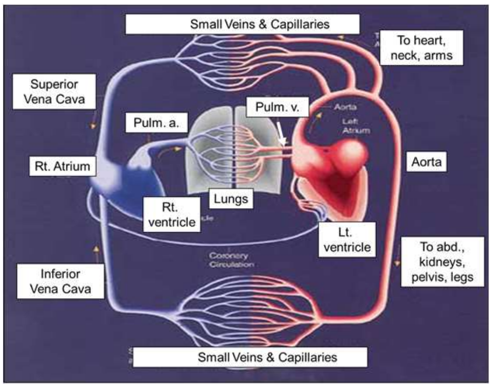
b) sinkt

c) bleibt gleich?

Und wenn der Teich sehr groß ist ?

Exercise from the lecture TH_Hydrostatics

Ex 1. Pipe flow



Input data:

- $\rho = 1060$ [kg m^{-3}]
- $\mu = 3.0 \times 10^{-3}$ [kg m^{-1} s^{-1}]
- Heart pumps about 6 liter per minute
- Diameter of aorta: 0.025 [m]

Simplifications:

- Consider blood as Newtonian fluid
- Neglect pulsating flow character; approximate peak discharge as twice average discharge (i.e. as if 12 liter per minute were flowing at constant rate)
- Neglect the elasticity of the aorta

Questions:

- Compute the main flow characteristics: $-\partial p^*/\partial x$, τ_b, $u(r)$, u_{max}, U
- Is the flow laminar or turbulent ?
- What is the effect of a change in diameter ? Hint: express Re as a function of Q and D

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Exercise from the lecture TH_Hydrostatics. Questions 2 and 3 have been resolved in the lecture TH_Laminar_Turbulent.

Beispiel 1. Rohrstömung

$$Q = \frac{\pi}{128\mu} \cdot \left(-\frac{\partial p^*}{\partial x}\right) \cdot D^4$$

$$\Rightarrow \left(-\frac{\partial p^*}{\partial x}\right) = \frac{Q \cdot 128 \cdot \mu}{\pi \cdot D^4} = \frac{0.0002 \cdot 128 \cdot 3 \cdot 10^{-3}}{\pi \cdot 0.025^4} = 62.6\ \frac{N}{m^3}$$

$$\pi \cdot R^2 \cdot \frac{\partial p^*}{\partial x} = 2 \cdot \pi \cdot R \cdot \tau_b \Rightarrow \tau_b = \frac{\partial p^*}{\partial x} \cdot \frac{R}{2} = -62.6 \cdot \frac{0.025}{4} = -0.39\,\frac{N}{m^2}$$

$$u_{max} = u(r=0) = \left[\frac{1}{4 \cdot \mu} \cdot \left(\frac{\partial p^*}{\partial x}\right) \cdot R^2\right] = \left[\frac{1}{4 \cdot 3 \cdot 10^{-3}} \cdot 62.6 \cdot 0.025^2\right] = 0.82\ m/s$$

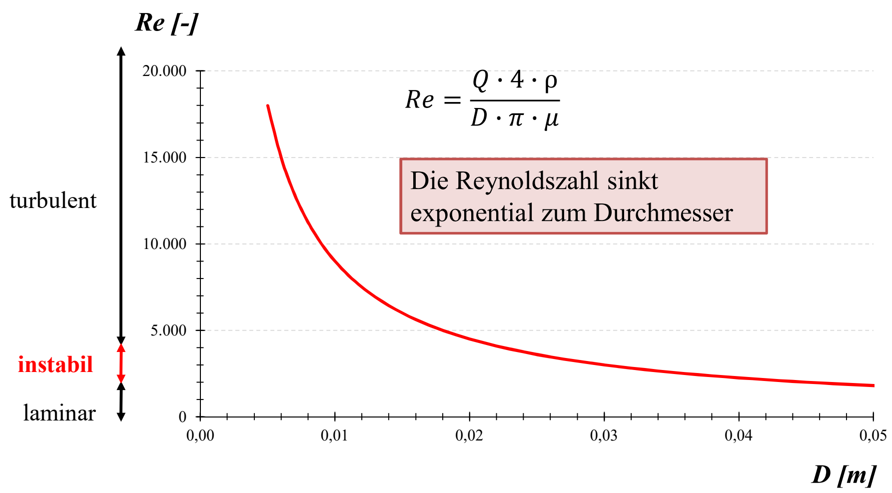
$$u(r) = u(r=0) \cdot \left(1 - \frac{r^2}{R^2}\right)$$

$$U = \frac{Q}{S} = \frac{Q}{\frac{D^2}{4} \cdot \pi} = \frac{0.0002}{\frac{0.025^2}{4} \cdot \pi} = 0.41 m/s$$

$$Re = \frac{U \cdot D \cdot \rho}{\mu} = \frac{0.41 \cdot 0.025 \cdot 1060}{3 \cdot 10^{-3}} = 3599$$

Re_{cr} 2000 – 4000 ⇒ instabil: Strömung kann laminar oder turbulent sein.
In einem gesunden Körper ist die Strömung unter normalen Umständen laminar!

Beispiel 1. Rohrstömung
Reynolds Zahl in Abhängigkeit vom Rohrdurchmesser



Ex 2. Pipe flow

Input data:

- $\rho = 900$ [kg m^{-3}]
- $\mu = 1.0$ [kg m^{-1} s^{-1}]. Note that the viscosity of different kinds of oil varies over at least two orders of magnitude. The viscosity can be lowered by adding additives.
- $Q = 2.5$ [m^3 s^{-1}]
- $D = 1$ [m]; the pipe material is very smooth

Simplifications:

- Consider oil as Newtonian fluid. In reality, oil is non-Newtonian with a viscosity that strongly depends on temperature and other factors

Questions:

- Compute the main flow characteristics: $-\partial p^*/\partial x$, τ_b, $u(r)$, u_{max}, U
- Is the flow laminar or turbulent ?
- What is the effect of a change in viscosity on the flow regime (Re) and the required pressure gradient ? Consider viscosities that are 10 times higher and lower.

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Exercise from the lecture TH_Hydrostatics. Questions 2 and 3 have been resolved in the lecture TH_Laminar_Turbulent.

Beispiel 2. Rohrströmung

$$Q = \frac{\pi}{128*\mu} \cdot \left(-\frac{\partial p^*}{\partial x}\right) \cdot D^4 \Rightarrow \left(-\frac{\partial p^*}{\partial x}\right) = \frac{Q \cdot 128 \cdot \mu}{\pi \cdot D^4} = \frac{2{,}5 \cdot 128}{\pi \cdot 1^4} = 101.86 \frac{N}{m^3}$$

$$\pi \cdot R^2 \cdot \frac{\partial p^*}{\partial x} = 2 \cdot \pi \cdot R \cdot \tau_b \;\Rightarrow\; \tau_b = \frac{\partial p^*}{\partial x} \cdot \frac{R}{2} = -101{,}86 \cdot \frac{1}{4} = -25.46 \frac{N}{m^2}$$

$$u(r) = \left(\frac{1}{4 \cdot \mu} \cdot \left(-\frac{\partial p^*}{\partial x}\right) \cdot R^2\right) \cdot \left(1 - \frac{r^2}{R^2}\right)$$

$$u_{max} = u(r = 0) = \left(\frac{1}{4 \cdot \mu} \cdot \left(-\frac{\partial p^*}{\partial x}\right) \cdot R^2\right) = \frac{1}{4 \cdot 1} \cdot 101.86 \cdot \frac{1^2}{4} = 6.37 \frac{m}{s}$$

$$U = \frac{Q}{S} = \frac{Q}{R^2 \cdot \pi} = \frac{2.5}{\frac{1^2}{4} \cdot \pi} = 3.18 \frac{m}{s}$$

Beispiel 2. Rohrströmung

Ist die Strömung laminar oder turbulent?

$$Re = \frac{U \cdot D}{\nu} \qquad \nu = \frac{\mu}{\rho}$$

$$Re = \frac{3.18 \cdot 1}{\frac{1}{900}} = 2865$$ ***Die Strömung ist instabil***

Welchen Effekt hat eine Änderung der Viskosität auf die Strömung (Re) und den Druckgradienten?

$$\boldsymbol{\mu = 10}: \left(-\frac{\partial p^*}{\partial x}\right) = \frac{Q \cdot 128 \cdot \mu}{\pi \cdot D^4} = \frac{2.5 \cdot 128 \cdot 10}{\pi \cdot 1^4} = 1018.6 \frac{N}{m^3}$$

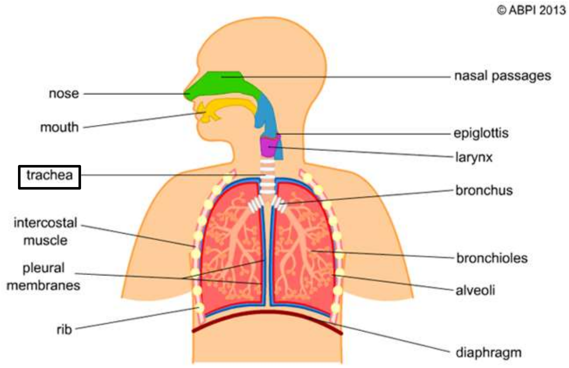
$$Re = \frac{3.18 \cdot 1}{\frac{10}{900}} = 286.5$$ ***Die Strömung ist laminar***

$$\boldsymbol{\mu = 0,1}: \left(-\frac{\partial p^*}{\partial x}\right) = \frac{Q \cdot 128 \cdot \mu}{\pi \cdot D^4} = \frac{2.5 \cdot 128 \cdot 0,1}{\pi \cdot 1^4} = 10.18 \frac{N}{m^3}$$

$$Re = \frac{3.18 \cdot 1}{\frac{0.1}{900}} = 28648$$ ***Die Strömung ist turbulent***

Beispiel 3. Rohrströmung

Case	ρ [kg m^{-3}]	μ [kg m^{-1} s^{-1}]	$\nu=\mu/\rho$ [m^2 s^{-1}]	U [m s^{-1}]	D [m]	Re [-]
Respiration	1.1	1.87E-05	1.70E-05	1.6	0.02	1.9E+03



Assume that the air flow in the trachea can be approximated by flow in a rigid pipe.

1. Compute and plot the energy losses per unit length as a function of the air velocity for a diameter of 0.02 m.
2. Compute and plot the energy losses per unit length as a function of the diameter for a given discharge corresponding to the values given in the table.

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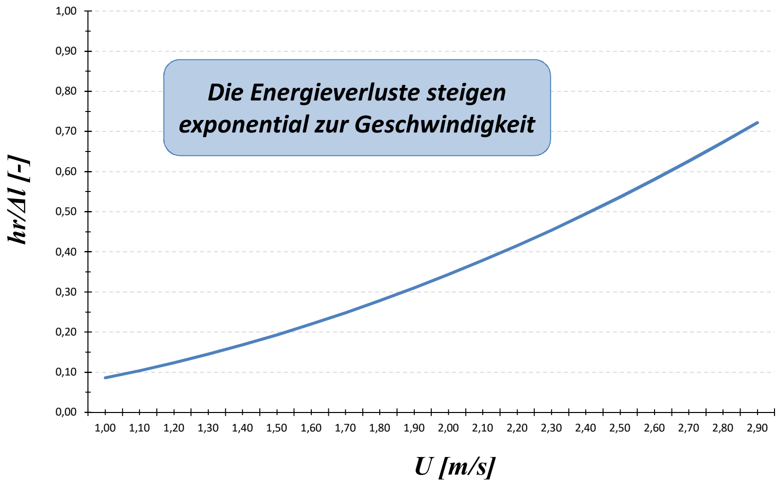
Example from the lecture TH_Laminar_Turbulent. To-be-developed similar to the example given in the lecture for arterial flow

Was passiert, wenn wir die Luftgeschwindigkeit ändern?

Case	ρ [kg m^{-3}]	μ [kg m^{-1} s^{-1}]	$\nu=\mu/\rho$ [m^2 s^{-1}]	U [m s^{-1}]	D [m]	Re [-]
Respiration	1.1	1.87E-05	1.70E-05	x	0.02	1.9E+03



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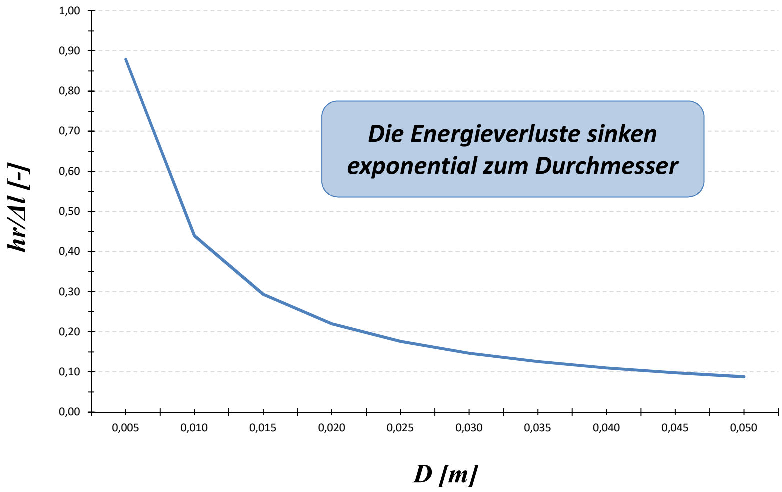
Was passiert, wenn wir den Durchmesser ändern?

Case	ρ [kg m^{-3}]	μ [kg m^{-1} s^{-1}]	$\nu=\mu/\rho$ [m^2 s^{-1}]	U [m s^{-1}]	D [m]	Re [-]
Respiration	1.1	1.87E-05	1.70E-05	1.6	x	1.9E+03



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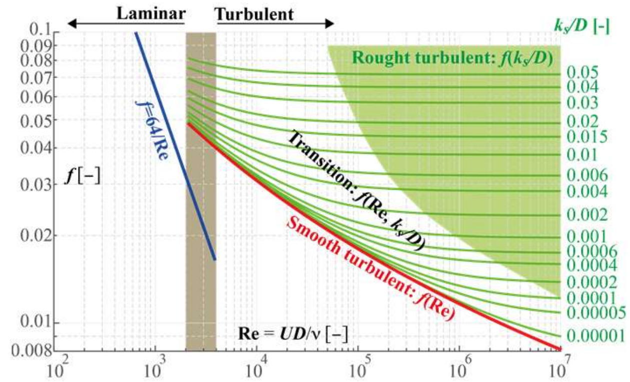
Ex 4. Pipe flow

Water at a temperature of 10° flows in a pipe of diameter D = 0.3 m. The roughness of the steel pipe is characterized by an equivalent sand roughness of k_s = 0.0003 m. The energy losses per unit length are 0.002.

1) Determine the flow regime
2) Determine the friction Dary-Weisbach friction coefficient f
3) Determine the discharge Q

Hint: the solution makes use of the Moody-Stanton diagram or the equivalent Colebrook-White formula



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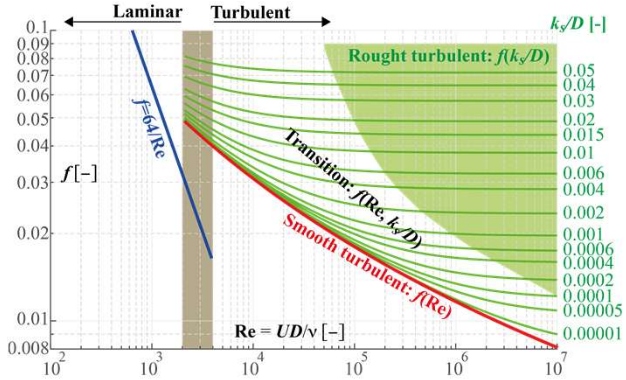
Ex 5. Pipe flow

A 0.250 $m^3 s^{-1}$ discharge of petrol with a kinematic viscosity of 9 x 10-6 $m^2 s^{-1}$ is transported in a 10'000 m long steel pipeline with characteristic sand roughness k_s = 0.00005 m. The total energy loss is 25 m.

1) Determine the flow regime
2) Determine the friction Darcy-Weisbach friction coefficient f
3) Determine the diameter of the pipe.

Hint: the solution makes use of the Moody-Stanton diagram or the equivalent Colebrook-White formula



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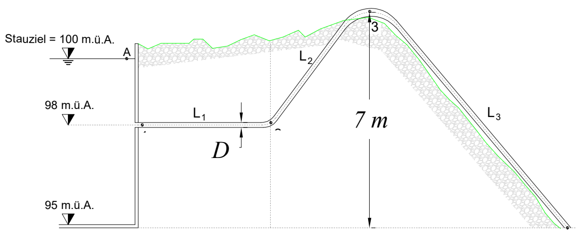
Ex 6. Pipe flow

Siphon: A tube used to convey liquid upwards from a reservoir and then down to a lower level of its own accord. Once the liquid has been forced into the tube, typically by suction or immersion, flow continues unaided



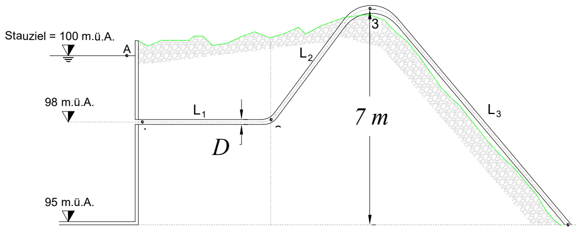
D = 0.5 m
k_s = 0.00015 m
$L_1 = L_2$ = 4 m, L_3 = 8m

Case 1: All minor energy losses are neglected.

1) Determine the discharge *Q*, the Darcy-Weisbach friction coefficient f, and the flow regime.
2) Draw the total energy line, and the potential energy line. Deduce from both the evolution of the pressure along the pipe.
3) Where does the minimum pressure in the pipe occur and what is its value ?
4) Is there a risk of cavitation in that point ?

Ex 7. Pipe flow

Siphon: A tube used to convey liquid upwards from a reservoir and then down to a lower level of its own accord. Once the liquid has been forced into the tube, typically by suction or immersion, flow continues unaided



D = 0.5 m
k_s = 0.00015 m
L_1 = L_2 = 4 m, L_3 = 8m

Case 2: Minor energy losses occur at the pipe inflow (K = 0.2), and in the two bends (K = 0.3 for each bend)

1) Determine the discharge *Q*, the Darcy-Weisbach friction coefficient f, and the flow regime.
2) Draw the total energy line, and the potential energy line. Deduce from both the evolution of the pressure along the pipe.
3) Where does the minimum pressure in the pipe occur and what is its value ?
4) Is there a risk of cavitation in that point ?