VORLESUNG

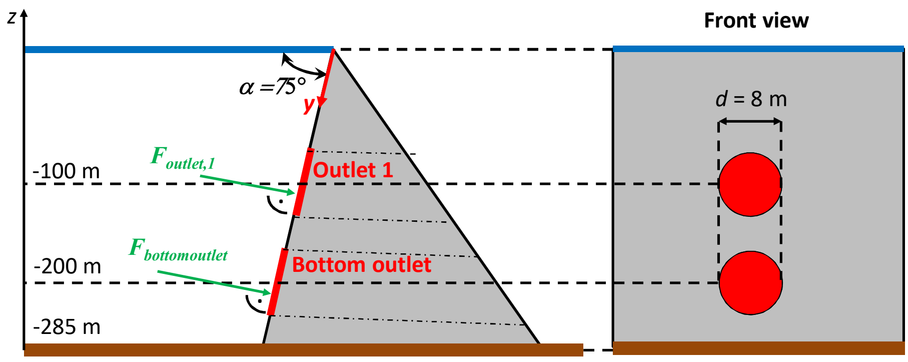
TECHNISCHE HYDRAULIK

222.564

Exercises - Solutions

Hydrostatics – Laminar Flow – Turbulent Flow – Pipe Flow

Ex 1. Hydrostatics. Forces on submerged inclined planes



$$I_{\xi\xi} = \frac{1}{64}\pi d^4$$

What are the magnitude, direction and application point of the forces acting on both outlet gates ?

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Exercise from the lecture TH_Hydrostatics

Ex 1. Hydrostatics. Solution

- Magnitude of the force: $F = \gamma \sin\alpha \, y_S A = \gamma h_S A$

  With h_s the depth below the water surface of the centre of gravity of the outlet valves

  $F_{outlet,1}$ = 4.93 x 10^7 [N]
  $F_{bottomoutlet}$ = 9.86 x 10^7[N]

- Direction of the force: perpendicular to the outlets = at an angle of 75° with the vertical
- Application point of the force, h_D: $y_D - y_s = \frac{I_{\xi\xi}}{y_S A} \longrightarrow h_D - h_s = \frac{I_{\xi\xi}}{y_s A}\sin\alpha = \frac{I_{\xi\xi}}{h_s A}\sin^2\alpha$

  $h_{D,outlet,1}$ = 100.037 [m]
  $h_{D,bottomoutlet}$ = 200.019 [m]

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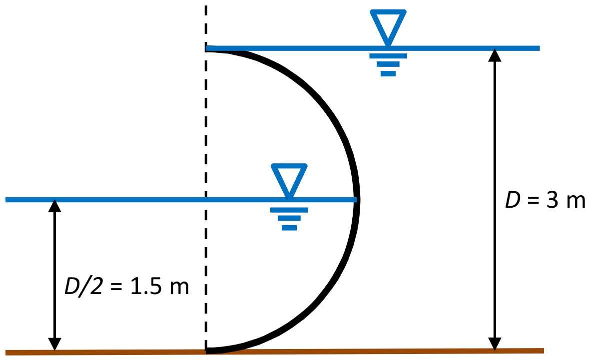
The solution is found by applying the relevant formulae seen in TH_Hydrostatics

Ex 2. Hydrostatics. Forces on submerged curved planes



1) Draw the pressure distribution and force components on the structure
2) Compute the horizontal force component, the vertical force component, as well as the resultant force, its direction and its action point.

Note that all considerations are per unit width

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Exercise from the lecture TH_Hydrostatics

Ex 2. Hydrostatics. Solution

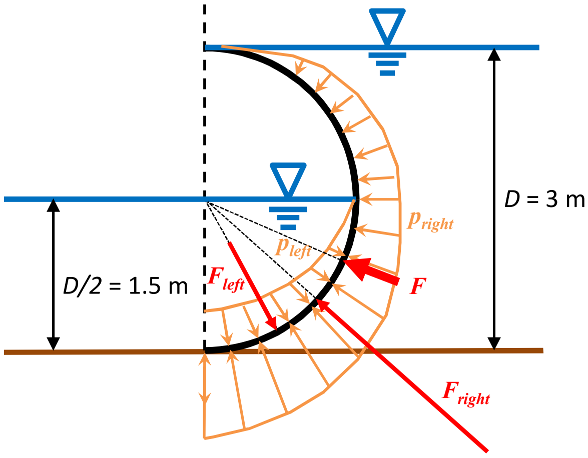
1) Draw the pressure distribution and force components on the structure

Pressure distribution: $p = \rho g h$, where

- h is the vertical distance below the water surface
- p acts perpendicularly to the structure

The resultant force components at both sides also act perpendicularly to the structure.

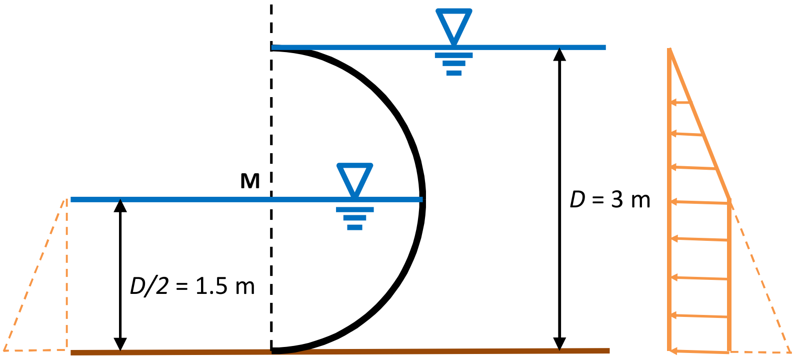
The resultant of forces at both sides (F) also acts perpendicularly to the structure



Ex 2. Hydrostatics. Solution

2) Compute the horizontal force component, the vertical force component, as well as the resultant force, its direction and its action point.

- The easiest method to compute the resultant force on the structure is by separately computing the horizontal and vertical components.
- The horizontal component is equal to the horizontal pressure force that would act on the projection of the structure on a vertical plane.



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Ex 2. Hydrostatics. Solution

In general, $F_{hor} = \rho g h_s A$, where h_s is the depth of the centre of gravity below the free surface, and A is the surface area on which the pressure acts

- Here, $h_{s,left} = D/4$, and $h_{s,right} = D/2$
- Since our considerations are per unit with, $A_{left} = D/2$ and $A_{right} = D$

$$F_{hor,left} = \frac{\rho g D^2}{8} = 11.04 \text{ [kN m}^{-1}\text{]}$$

$$F_{hor,right} = \frac{\rho g D^2}{2} = 44.15 \text{ [kN m}^{-1}\text{]}$$

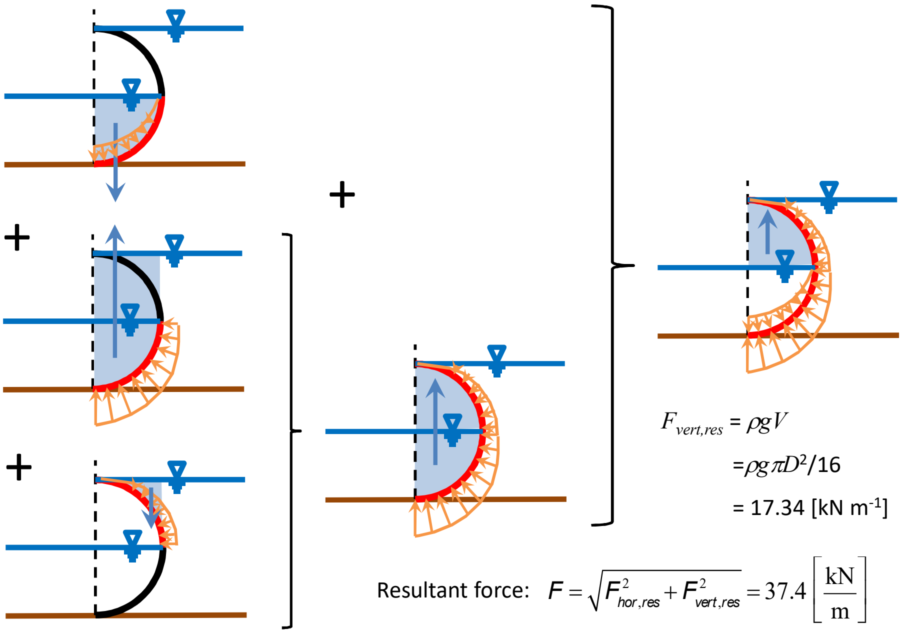
$F_{hor,res}$ = 33,11 [kN m^{-1}], acting from right to left

It is straightforward to compute the action point of F_{hor}. In order to determine the action point of the resultant of F_{hor} and F_{vert}, however, it is more convenient to exploit the characteristic that the resultant is perpendicular to the structure.

2) Compute the horizontal force component, the vertical force component, as well as the resultant force, its direction and its action point.

- The vertical component is equal to the weight of the fluid above the structure, which is not trivial in this case. A convenient approach consists in dividing the contributions to the vertical force into three parts:

Ex 2. Hydrostatics. Solution



$F_{vert,res} = \rho g V$

$= \rho g \pi D^2/16$

$= 17.34$ [kN m^{-1}]

Resultant force: $F = \sqrt{F_{hor,res}^2 + F_{vert,res}^2} = 37.4 \left[\frac{\text{kN}}{\text{m}}\right]$

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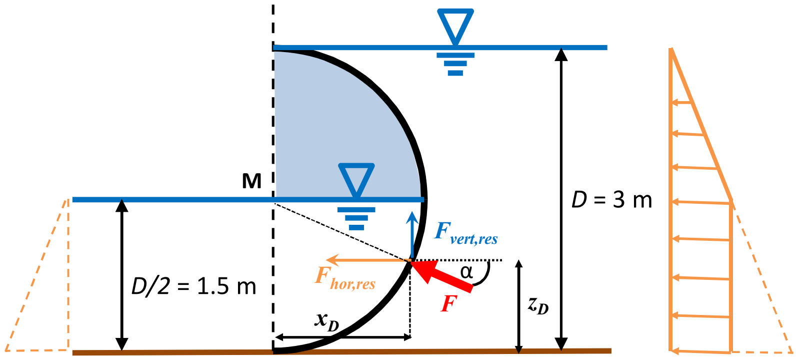
Ex 2. Hydrostatics. Solution



- Force direction: $\alpha = \text{atan}\left(\frac{F_{vert,res}}{F_{hor,res}}\right) = \text{atan}\left(\frac{17.34}{33.11}\right) = 27.64°$
- Force action point: the resultant force is perpendicular to the structure, and therefore goes through the circle centre:

$$\begin{cases} x_D = \frac{D}{2}\cos\alpha = 1.33\ [m] \\ z_D = \frac{D}{2}(1-\sin\alpha) = 0.80\ [m] \end{cases}$$

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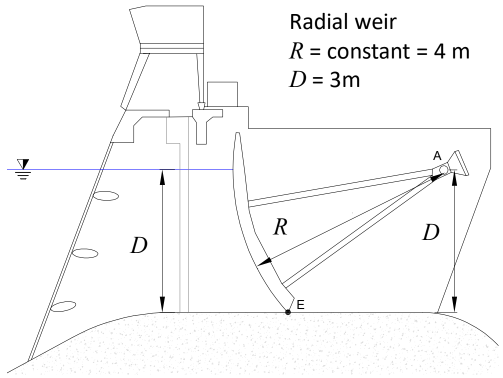
Ex 3. Hydrostatics. Forces on submerged curved planes



Radial weir
R = constant = 4 m
D = 3m

1) Draw the pressure distribution and force components on the structure
2) Compute the horizontal force component, the vertical force component, as well as the resultant force, its direction and its action point.

Note that all considerations are per unit width

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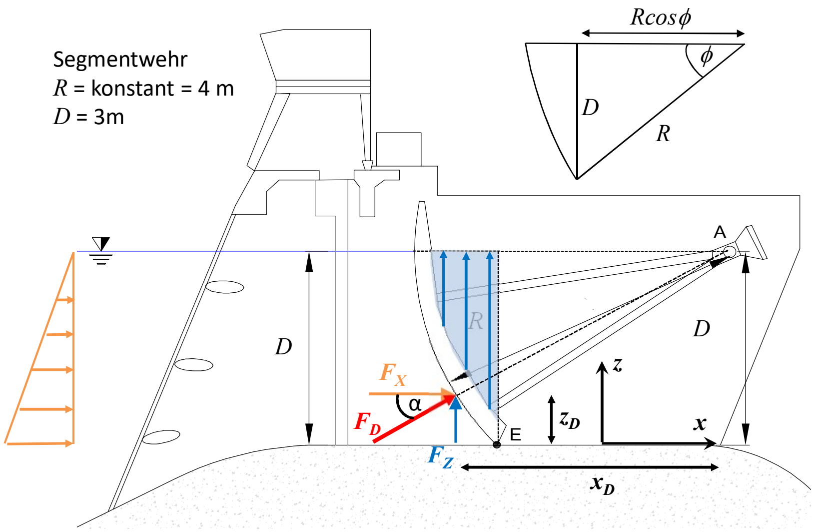
Exercise from the lecture TH_Hydrostatics

Ex 3. Hydrostatics. Solution

1) Draw the pressure distribution and force components on the structure



Exercise from the lecture TH_Hydrostatics

Ex 3. Hydrostatics. Solution

2) Compute the horizontal force component, the vertical force component, as well as the resultant force, its direction and its action point.

- The easiest method to compute the resultant force on the structure is by separately computing the horizontal and vertical components.
- The horizontal component is equal to the horizontal pressure force that would act on the projection of the structure on a vertical plane.
- In general, $F_x = \rho g h_s A$, where h_s is the depth of the centre of gravity below the free surface, and A is the surface area on which the pressure acts. Here, $h_s = D/2$. Since our considerations are per unit with, $A = D$.

$$F_x = \frac{\rho g D^2}{2} = 44.15 \left[\frac{\text{kN}}{\text{m}}\right]$$

- The vertical component F_z is equal to the weight of the fluid above the structure (indicated in the sketch). It obviously acts in upward direction in this case.

$$F_z = \rho g V = \rho g \left(\pi R^2 \frac{\phi}{360} - \frac{R\cos\phi . D}{2} \right) = 27.6 \left[\frac{\text{kN}}{\text{m}}\right] \text{, where } \sin\phi = \frac{3}{4} \rightarrow \phi = 48.6°$$

- The resultant force is: $F_D = \sqrt{F_x^2 + F_z^2} = 52.1 \left[\frac{\text{kN}}{\text{m}}\right]$

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The solution is found by applying the relevant formulae seen in TH_Hydrostatics

Ex 3. Hydrostatics. Solution

2) Compute the horizontal force component, the vertical force component, as well as the resultant force, its direction and its action point.

- Because the pressure acts perpendicularly to the circular structure in every point, the resultant force will also act perpendicularly to the structure, and its action line will pass through the circle centre.
- Force direction: $\alpha = \text{atan}\left(\frac{F_z}{F_x}\right) = \text{atan}\left(\frac{27.62}{44.15}\right) = 32.0°$
- Force action point: the resultant force is perpendicular to the structure, and therefore goes through the circle centre:

$$\begin{cases} x_D = R\cos\alpha = 3.39\ [m] \\ z_D = D - R\sin\alpha = 0.88\ [m] \end{cases}$$

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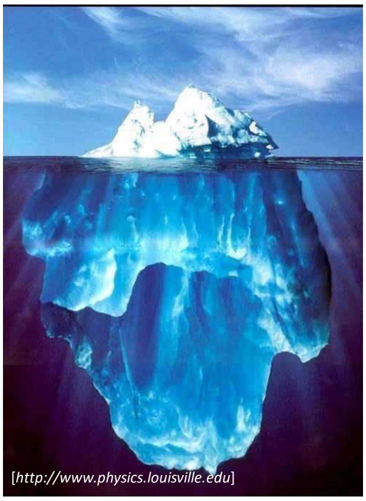
The solution is found by applying the relevant formulae seen in TH_Hydrostatics

Ex 4. Hydrostatics. Archimedes principle – buoyant force



ρ_{water} (0°) = 999.8 kg m^{-3}
ρ_{ice} (0°) = 916.7 kg m^{-3}

What percentage volume of the iceberg sticks out above the water surfce ?

Exercise from the lecture TH_Hydrostatics

Ex 4. Hydrostatics. Solution

The downward and upward forces are in equilibrium: $F_{down} = F_{up}$

The downward force is the weight of the iceberg: $F_{down} = \rho_{ice} g V_{ice}$

The upward Archimedes force is equal to the weight of the displaced water: $F_{up} = \rho_{water} g V_{ice,immersed}$

$$\rightarrow \frac{V_{ice,immersed}}{V_{ice}} = \frac{\rho_{ice}}{\rho_{water}} = \frac{916.7}{999.8} = 0.92$$

→ 8% of the iceberg sticks out of the water

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The solution is found by applying the relevant formulae seen in TH_Hydrostatics

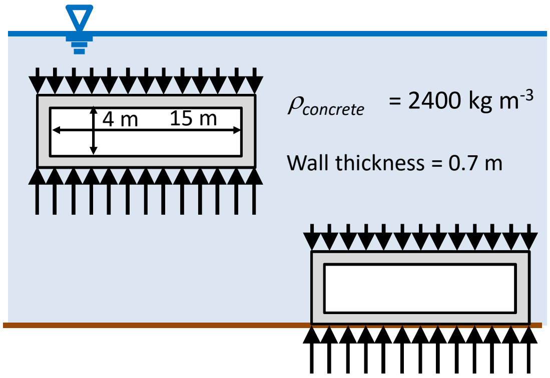
Ex 5. Hydrostatics. Archimedes principle – buoyant force

1) Will the tunnel element float or sink ?

2) If it floats, what force is required to bring it down to the bottom ?

3) If it floats, what is its stable position (by how much does it stick out of the water) ?

4) What is the resulting vertical force per unit length when the tunnel element lays on the bottom?



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Exercise from the lecture TH_Hydrostatics

Ex 5. Hydrostatics. Solution

- **1) Will the tunnel element float or sink ?**

Let us assume that the tunnel element is totally immersed in the water. If its downward weight W is smaller/larger than than the upward Archimedes force F_A, it will float/sink. Note that we consider forces per unit length.

$$W = \rho_{concrete} g V_{concrete}$$
$$= 2400 \text{ x } 9.81 \text{ x } [(4 + 2\text{x}0.7) \text{ x } (15 + 2\text{x}0.7) - 4\text{x}15]$$
$$= 672.4 \text{ [kN m}^{-1}\text{]}$$

$$F_A = \rho_{water} g V_{displacedfluid}$$
$$= 1000 \text{ x } 9.81 \text{ x } [(4 + 2\text{x}0.7) \text{ x } (15 + 2\text{x}0.7)]$$
$$= 868.8 \text{ [kN m}^{-1}\text{]}$$

$F_A > W$
→ the tunnel element floats

2) If it floats, what force is required to bring it down to the bottom ?

To bring it to the bottom, an additional downward force $\boldsymbol{F}$ is required, such that the total downward force is larger than the upward Archimedes force:

$$W + F > F_A \rightarrow F > F_A - W = 868.8 - 672.4 = 196.4 \text{ [kN m}^{-1}\text{]}$$

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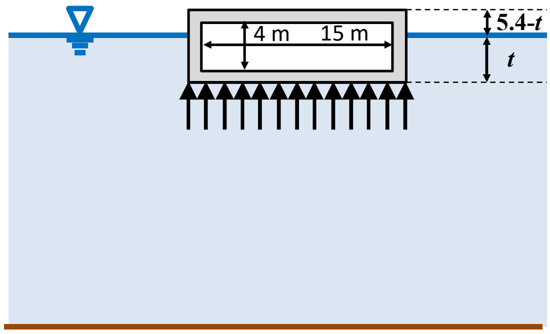
Ex 5. Hydrostatics. Solution

3) If it floats, what is its stable position (by how much does it stick out of the water) ?



The tunnel element floats when $W = F_A$. The weight W does not depend on the degree of immersion, but because the volume of displaced fluid depends on it, F_A also depends on the degree of immersion.

$W = 672.4$ [kN m^{-1}]

$F_A = \rho_{water} g V_{displacedfluid}$

$= 1000 \times 9.81 \times [t \times (15 + 2 \times 0.7)]$

$= 160.9 \times t$ [kN m^{-1}]

$W = F_A$ if $t = 4.18$ [m]

→ The tunnel element sticks $5.4 - t = 1.22$ [m] out of the water

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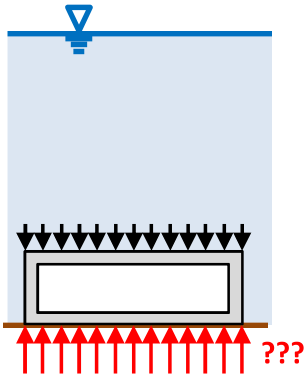
Ex 5. Hydrostatics. Solution

4) What is the resulting vertical force per unit length when the tunnel element lays on the bottom?



This is a tricky question. It all depends on the pressure forces on the bottom of the tunnel element.

As long as there is water below the tunnel element, the hydrostatic pressure will act on the bottom and the Archimedes force F_A computed in question 1) will tend to lift the tunnel element from the bottom. This will be the case if:

- The bottom consists of granular material, because there will be water in the pores between the grains.
- The bottom is not perfectly flat. It is very difficult in that case to expulse all the water situated below the tunnel element.

Only in case of a perfectly flat and impermeable bottom surface, there will not be any hydrostatic pressure anymore on the bottom of the tunnel element. In that case, the only vertical component of the hydrostatic pressure is on the top of the tunnel element and it stabilizes the tunnel element on the bottom.

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Ex 6. Hydrostatics. Archimedes principle – buoyant force

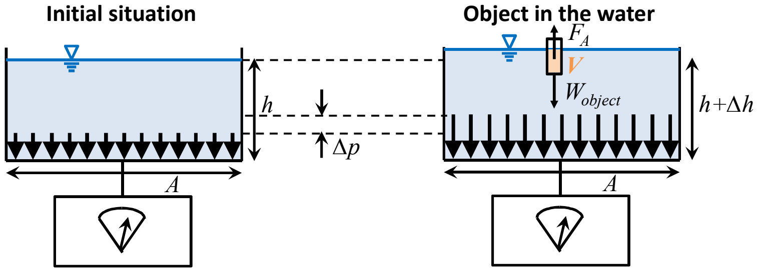
Ein Behälter mit Wasser steht auf einer Waage. Man taucht den Finger ein – was passiert mit der Anzeige auf der Waage?

a) steigt
b) sinkt
c) bleibt gleich?

Begründung ?

Exercise from the lecture TH_Hydrostatics

Ex 6. Hydrostatics. Solution



The weight indicated by the balance is the resultant of the vertical component of all pressure contributions: $W=pA=\rho ghA$

Assume that the inserted object has an immersed volume V. The water level in the recipient will rise by $\Delta h=V/A$, and the pressure on the bottom will rise by $\Delta p=\rho g\Delta h$. Hence, the integral of the pressure on the bottom will rise by $\Delta W=\Delta pA=\rho gV$.

- In case the inserted object floats, $\Delta W = \rho gV = F_A$ is equal to the upward Archimedes force on the object (weight of displaced fluid), which is equal to the total weight of the object, W_{object}. So we find the very logical result that the weight indicated by the balance is the weight of the water plus the weight of the inserted object.
- But when the inserted object is your finger, it does not float because you carry your finger. This means that only the upward Archimedes force acts, and it directly balances the additional pressure force on the bottom ΔW. Hence the weight indicated by the balance will not change.

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Ex 7. Hydrostatics. Archimedes principle – buoyant force

Ein Fischer sitzt in einem Boot in einem kleinen Teich und wirft den Anker aus. Was passiert mit dem Wasserspiegel?

a) steigt

b) sinkt

c) bleibt gleich?

Und wenn der Teich sehr groß ist ?

Exercise from the lecture TH_Hydrostatics

Ex 7. Hydrostatics. Solution

When the anchor is thrown in the water:

- It displaces a volume of fluid in the pond that is equal to its own volume, $\Delta V_1 = V_{anchor}$.
- The boat becomes lighter by $\Delta W = \rho_{anchor} V_{anchor}$. As a result, the Archimedes force required to keep the boat floating is reduced by $\Delta F_A = \Delta W = \rho_{water} \Delta V_2$, where ΔV_2 represents the corresponding reduction in the volume of displaced fluid.

→ The total volume of displaced fluid in the pond will decrease by:

$$\Delta V_2 - \Delta V_1 = V_{anchor} \left(\frac{\rho_{anchor}}{\rho_{water}} - 1 \right) > 0$$

This causes a lowering of the water level in the pond by $\Delta h = (\Delta V_2 - \Delta V_2) / A$, where A is the surface area of the pond. This lowering is negligible, even for very small ponds.

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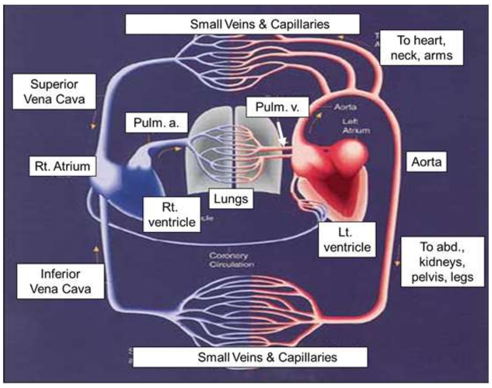
The solution is found by applying the relevant formulae seen in TH_Hydrostatics

Ex 1. Pipe flow



Input data:

- $\rho = 1060$ [kg m^{-3}]
- $\mu = 3.0 \times 10^{-3}$ [kg m^{-1} s^{-1}]
- Heart pumps about 6 liter per minute
- Diameter of aorta: 0.025 [m]

Simplifications:

- Consider blood as Newtonian fluid
- Neglect pulsating flow character; approximate peak discharge as twice average discharge (i.e. as if 12 liter per minute were flowing at constant rate)
- Neglect the elasticity of the aorta

Questions:

- Compute the main flow characteristics: $-\partial p^*/\partial x$, τ_b, $u(r)$, u_{max}, U
- Is the flow laminar or turbulent ?
- What is the effect of a change in diameter ? Hint: express Re as a function of Q and D

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Exercise from the lecture TH_LaminarFlow. Questions 2 and 3 have been resolved in the lecture TH_Laminar_Turbulent.

Ex 1. Pipe flow. Solution

Let us assume that the flow is laminar, which is to be verified !

- **Hagen-Poiseuille law:** $Q = \frac{\pi}{8\mu}\left(-\frac{\partial p^*}{\partial x}\right)R^4 = \frac{\pi}{128\mu}\left(-\frac{\partial p^*}{\partial x}\right)D^4 \rightarrow -\frac{\partial p^*}{\partial x} = 62.6\left[\frac{\mathrm{N}}{\mathrm{m}^3}\right]$

  Note: it is important always to use SI units. This means that the discharge has to be converted in [m^3 s^{-1}]. It is good practice to verify that the units of the end result are correct.

- **Wall shear stress:** $\pi R^2 \frac{\partial p^*}{\partial x} = 2\pi R \tau_b \quad \rightarrow \quad \tau_b = -0.39\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$

  Note: Minus sign because the shear exerted by the wall on the flow is a resistance, which is opposed to the flow. The flow exerts a shear on the wall that of equal magnitude but opposite sign.

- **Velocity distribution:**

$$u = u(r=0)\left(1-\frac{r^2}{R^2}\right) = \left[\frac{1}{4\mu}\left(-\frac{\partial p^*}{\partial x}\right)R^2\right]\left(1-\frac{r^2}{R^2}\right) = 0.82\left(1-\frac{r^2}{R^2}\right) = u_{\max}\left(1-\frac{r^2}{R^2}\right)$$

$$U = \frac{Q}{S} = \frac{4Q}{\pi D^2} = 0.41\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$$

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The solutions are found by substituting the values in the relevant equations given in TH_Hydrostatics

Ex 1. Pipe flow: Solution

- **Flow regime:** $\mathrm{Re} = \frac{UD}{\nu} = \frac{\rho UD}{\mu} = 3599$

According to theory, the critical Reynolds number for the transition from laminar to turbulent flow is in the range 2000 to 4000 and depends on the specific flow configuration. The value obtained is therefor not conclusive. But nature is a good fluid mechanical engineer. Because laminar flow is energetically advantageous, arterial flow in a healthy person is laminar.

- **What is the effect of a change in diameter ? Hint: express Re as a function of Q and D**

The answer has been detailed in the lecture TH_Laminar_Turbulent

Ex 2. Pipe flow

Input data:

- $\rho = 900$ [kg m^{-3}]
- $\mu = 1.0$ [kg m^{-1} s^{-1}]. Note that the viscosity of different kinds of oil varies over at least two orders of magnitude. The viscosity can be lowered by adding additives.
- $Q = 2.5$ [m^3 s^{-1}]
- $D = 1$ [m]; the pipe material is very smooth

Simplifications:

- Consider oil as Newtonian fluid. In reality, oil is non-Newtonian with a viscosity that strongly depends on temperature and other factors

Questions:

- Compute the main flow characteristics: $-\partial p^*/\partial x$, τ_b, $u(r)$, u_{max}, U
- Is the flow laminar or turbulent ?
- What is the effect of a change in viscosity on the flow regime (Re) and the required pressure gradient ? Consider viscosities that are 10 times higher and lower.

Exercise from the lecture TH_LaminarFlow. Questions 2 and 3 have been resolved in the lecture TH_Laminar_Turbulent.

Ex 2. Pipe flow. Solution

Let us assume that the flow is laminar, which is to be verified !

- **Hagen-Poiseuille law:** $Q=\frac{\pi}{8\mu}\left(-\frac{\partial p^*}{\partial x}\right)R^4=\frac{\pi}{128\mu}\left(-\frac{\partial p^*}{\partial x}\right)D^4 \rightarrow -\frac{\partial p^*}{\partial x}=101.9\left[\frac{\mathrm{N}}{\mathrm{m}^3}\right]$

Note: it is important always to use SI units. This means that the discharge has to be converted in [m^3 s^{-1}]. It is good practice to verify that the units of the end result are correct.

- **Wall shear stress:** $\pi R^2\frac{\partial p^*}{\partial x}=2\pi R\tau_b \rightarrow \tau_b=-25.5\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$

Note: Minus sign because the shear exerted by the wall on the flow is a resistance, which is opposed to the flow. The flow exerts a shear on the wall that of equal magnitude but opposite sign.

- **Velocity distribution:**

$$u=u(r=0)\left(1-\frac{r^2}{R^2}\right)=\left[\frac{1}{4\mu}\left(-\frac{\partial p^*}{\partial x}\right)R^2\right]\left(1-\frac{r^2}{R^2}\right)=6.4\left(1-\frac{r^2}{R^2}\right)=u_{\max}\left(1-\frac{r^2}{R^2}\right)$$

$$U=\frac{Q}{S}=\frac{4Q}{\pi D^2}=3.2\left[\frac{\mathrm{m}}{\mathrm{s}^{-1}}\right]$$

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The solutions are found by substituting the values in the relevant equations given in TH_Hydrostatics

Ex 2. Pipe flow: Solution

- **Flow regime:** $\mathrm{Re} = \dfrac{UD}{\nu} = \dfrac{\rho UD}{\mu} = 2865$

According to theory, the critical Reynolds number for the transition from laminar to turbulent flow is in the range 2000 to 4000 and depends on the specific flow configuration. The value obtained is therefor not conclusive, and we can not be sure that the solutions obtained for $\partial p^*/\partial x$, τ_b and the velocity distribution are correct.

- **What is the effect of a change in viscosity ?**
  - The cross-sectionally averaged velocity will not change, because it is fully determined by the given Q and D.
  - For a viscosity that is 10 times higher, Re will be 10 times lower and the flow will be laminar. The velocity distribution derived on the previous slide will be correct. The flow will be induced by a value of $\partial p^*/\partial x$ that is 10 times higher than the one on the previous slide. The wall shear stress will also be 10 times higher than the one on the previous slide.
  - For a viscosity that is 10 times lower, Re will be 10 times higher and the flow will be turbulent. The solutions for laminar flow are not valid anymore, and the theory for turbulent flow has to be used.

Exercise 3: Pipe flow

Input data:

- $\rho = 1000$ [kg m^{-3}]
- $\mu = 1.0 \times 10^{-3}$ [kg m^{-1} s^{-1}]
- $Q = 8$ [m^3 s^{-1}]
- $D = 1$ [m]

Questions:

- Is the flow laminar or turbulent ?
- In case it is turbulent, would it be possible to make it laminar by changing D ?
- What slope would be required to transport Q in this pipe if the flow were laminar ?

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Exercise from TH_LaminarFlow

Ex 3. Pipe flow. Solution

- **Is the flow laminar or turbulent ?**

$$\left.\begin{aligned} \mathrm{Re} &= \frac{UD}{\nu} = \frac{\rho UD}{\mu} \\ U &= \frac{Q}{S} = \frac{4Q}{\pi D^2} \end{aligned}\right\} \quad \mathrm{Re} = \frac{UD}{\nu} = \frac{4\rho Q}{\pi\mu D} = 1.02\mathrm{x}10^{7} \longrightarrow \text{Turbulent}$$

- **In case it is turbulent, would it be possible to make it laminar by changing *D* ?**

$$D = \frac{4\rho Q}{\pi\mu \,\mathrm{Re}} = 5093\ [\mathrm{m}]$$,where Re = 2000 has been chosen to guarantee laminar flow

→ This is obviously an unrealistic value

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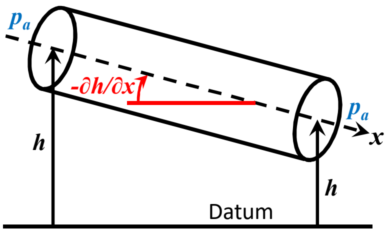
Ex 3. Pipe flow. Solution

- **What slope would be required to transport Q in this pipe if the flow were laminar ?**

Hagen-Poiseuille law: $Q = \frac{\pi}{8\mu}\left(-\frac{\partial p^*}{\partial x}\right) R^4 = \frac{\pi}{128\mu}\left(-\frac{\partial p^*}{\partial x}\right) D^4 \rightarrow -\frac{\partial p^*}{\partial x} = 0.33\left[\frac{\mathrm{N}}{\mathrm{m}^3}\right]$

$$-\frac{\partial p^*}{\partial x} = -\frac{\partial}{\partial x}\left(p+\rho g h\right) = -\frac{\partial p}{\partial x} - \rho g \frac{\partial h}{\partial x}$$

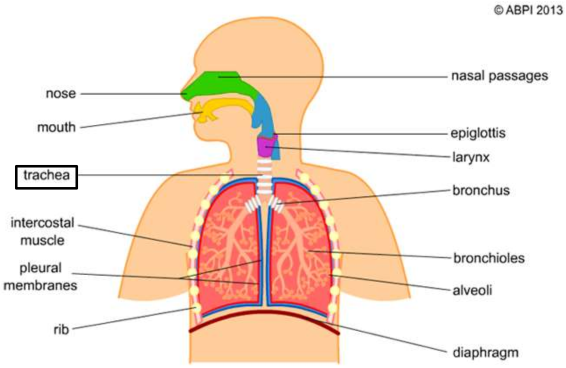
$$\rightarrow \text{slope} = \text{atan}\left(-\frac{\partial h}{\partial x}\right) \approx -\frac{\partial h}{\partial x} = 3.3\text{x}10^{-5}$$



A very mild slope would be sufficient to transport the discharge if the flow remained laminar at this Re number. Because the flow is turbulent at this Re number, the energy losses are much higher, and a much higher slope is required to transport the same discharge.

Ex 4. Pipe flow

Case	ρ [kg m^{-3}]	μ [kg m^{-1} s^{-1}]	$\nu=\mu/\rho$ [m^2 s^{-1}]	U [m s^{-1}]	D [m]	Re [-]
Respiration	1.1	1.87E-05	1.70E-05	1.6	0.02	1.9E+03



Assume that the air flow in the trachea can be approximated by flow in a rigid pipe.

1. Compute and plot the energy losses per unit length as a function of the air velocity for a diameter of 0.02 m.
2. Compute and plot the energy losses per unit length as a function of the diameter for a given discharge corresponding to the values given in the table.

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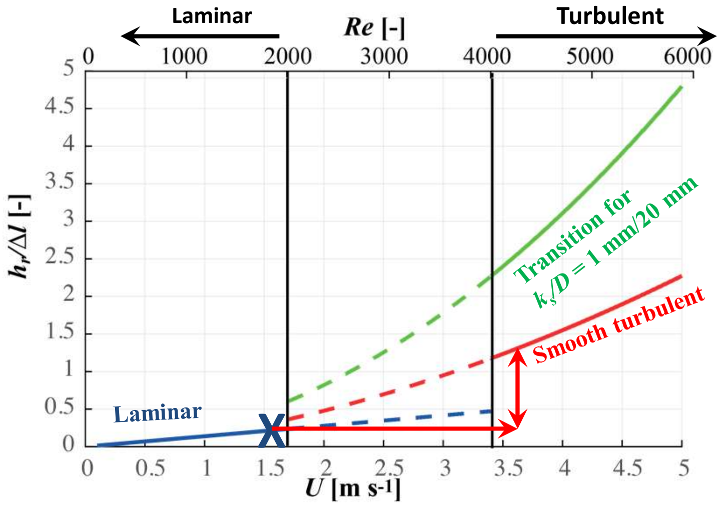
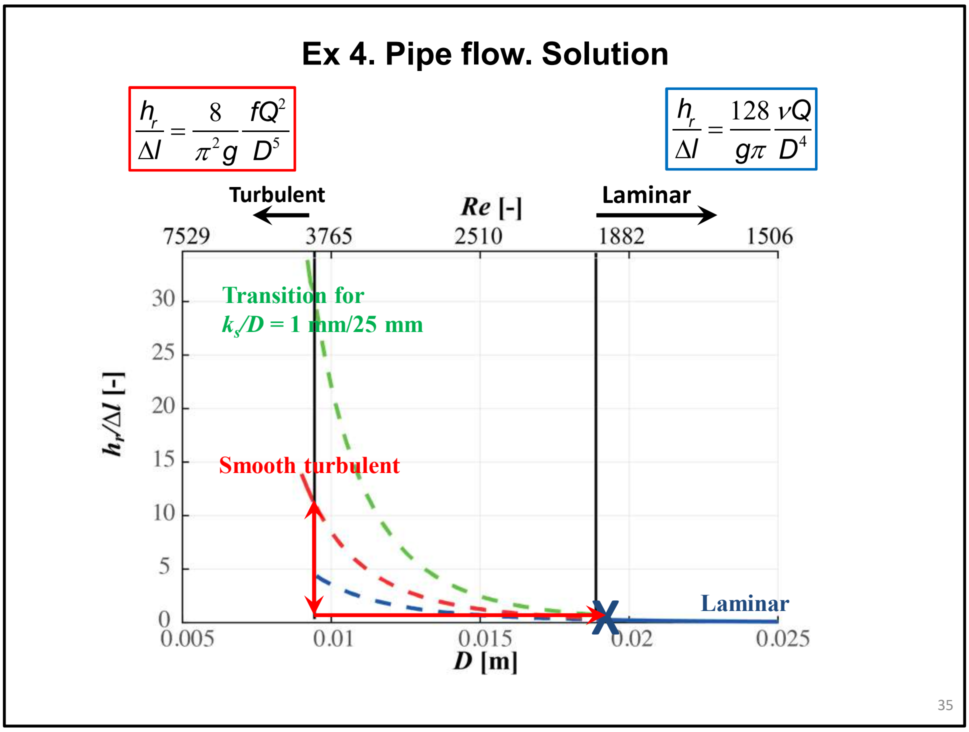
Example from the lecture TH_Laminar_Turbulent. The solution is similar to the example given in the lecture for arterial flow

Ex 4. Pipe flow. Solution

$$\frac{h_r}{\Delta l} = \frac{32}{g}\frac{\nu U}{D^2}$$

$$\frac{h_r}{\Delta l} = \frac{1}{2g}\frac{fU^2}{D}$$





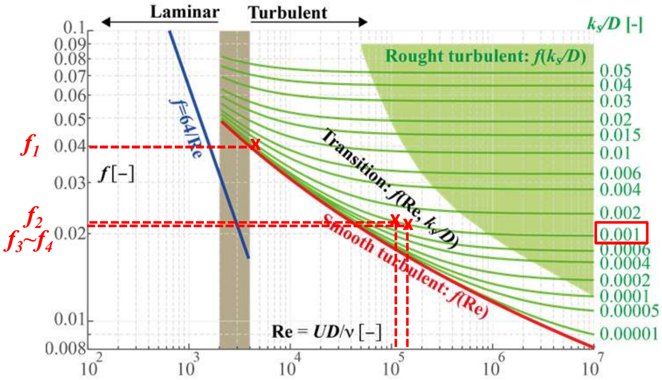
- Note that energy losses are much larger than in the arterial flow. This is because:
- (i) the air velocity in the trachea is much higher than the blood velocity in the aorta, leading to a higher discharge
- (ii) the kinematic viscosity of air is three times larger than that of water.

Ex 5. Pipe flow

Water at a temperature of 10° flows in a pipe of diameter D = 0.3 m. The roughness of the steel pipe is characterized by an equivalent sand roughness of k_s = 0.0003 m. The energy losses per unit length are 0.002.

1) Determine the flow regime
2) Determine the friction Dary-Weisbach friction coefficient f
3) Determine the discharge Q

Hint: the solution makes use of the Moody-Stanton diagram or the equivalent Colebrook-White formula



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Ex 5. Pipe flow. Solution

Solution according to procedure outlined in application example 2 of TH_Pipeflow

Water at 10°: $\nu = 1.307 \times 10^{-6}$ [m^2 s^{-1}]

D, h_r, k_s, ν known → Q, U, f, Re to be solved

Iteration step 1:

1. Initial guess of f.

$k_s = 0.3 \times 10^{-3}$ [m] → $k_s/D = 1 \times 10^{-3}$.

Re is unknown

} We choose an initial value of f on the k_s/D = 0.001 curve in the Moody-Stanton diagram

Since we expect for this flow a relatively high Re number, it would be logical to choose an initial value at the right side of the k_s/D = 0.001 curve close to the value for a rough turbulent flow that is independent of Re. **For the purpose of illustrating the iterative procedure**, let us take an initial value of f towards the left side of the k_s/D = 0.001 curve.

f_1 = 0.04 (see figure on previous slide).

2. Initial guess of Q from the Darcy-Weisbach equation: $Q_1 = \left(\frac{h_r}{\Delta l} \frac{\pi^2 g}{8} \frac{1}{f_1} D^5 \right)^{1/2} = 0.0383$ [m^3 s^{-1}]

3. Initial guess of Re: $\mathrm{Re}_1 = \frac{4Q_1}{\pi D \nu} = 1.25 \times 10^5$

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Ex 5. Pipe flow. Solution

Iteration step 2:

1. New guess of *f*.

$k_s = 0.3 \times 10^{-3}$ [m] → $k_s/D = 1 \times 10^{-3}$.

$\mathrm{Re}_1 = 1.72 \times 10^5$

} $f_2 = 0.022$ (see figure on previous slide)

2. New guess of Q from the Darcy-Weisbach equation: $Q_2 = \left(\frac{h_r}{\Delta l}\frac{\pi^2 g}{8}\frac{1}{f_2}D^5\right)^{1/2} = 0.0517$ [m³ s⁻¹]

3. New guess of *Re:* $\mathrm{Re}_2 = \frac{4Q_2}{\pi D \nu} = 1.68 \times 10^5$

Iteration step 3:

1. New guess of *f*.

$k_s = 0.3 \times 10^{-3}$ [m] → $k_s/D = 1 \times 10^{-3}$.

$\mathrm{Re}_2 = 1.68 \times 10^5$

} $f_3 = 0.021$ (see figure on previous slide)

2. New guess of Q from the Darcy-Weisbach equation $Q_3 = \left(\frac{h_r}{\Delta l}\frac{\pi^2 g}{8}\frac{1}{f_3}D^5\right)^{1/2} = 0.0529$ [m³ s⁻¹]

3. New guess of *Re:* $\mathrm{Re}_3 = \frac{4Q_3}{\pi D \nu} = 1.72 \times 10^5$

Iteration step 4:

1. New guess of *f*.

$k_s = 0.3 \times 10^{-3}$ [m] → $k_s/D = 1 \times 10^{-3}$.

$\mathrm{Re}_3 = 1.72 \times 10^5$

} $f_4 \approx f_3 = 0.021$ → Solution converged !

Flow regime: turbulent transition

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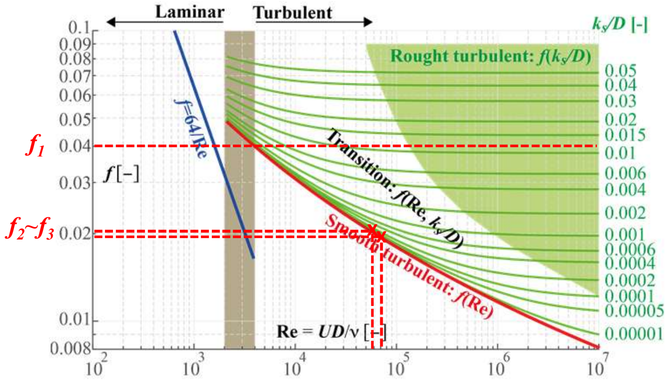
Ex 6. Pipe flow

A 0.250 $m^3 s^{-1}$ discharge of petrol with a kinematic viscosity of 9 x 10^{-6} $m^2 s^{-1}$ is transported in a 10'000 m long steel pipeline with characteristic sand roughness k_s = 0.00005 m. The total energy loss is 25 m.

1) Determine the flow regime
2) Determine the friction Darcy-Weisbach friction coefficient f
3) Determine the diameter of the pipe.

Hint: the solution makes use of the Moody-Stanton diagram or the equivalent Colebrook-White formula



Ex 6. Pipe flow. Solution

Q, h_r, k_s, ν known → U, D, f, Re to be solved

Iteration step 1:

1. Initial guess f_1. Both Re and k_s/D are unknown, rendering the guess more difficult.

   f_1 = 0.04 (see figure on previous slide).

2. Initial guess D_1 from Darcy-Weisbach equation: $D_1 = \left(\frac{8}{\pi^2 g}\frac{\Delta l Q^2}{h_r}\right)^{1/5} f_1^{1/5} = 0.61$ [m]

3. Initial guess Re_1: $Re_1 = \frac{4Q}{\pi D_1 \nu} = 5.8 \times 10^4$

Iteration step 2:

1. New guess of f: $k_s/D = 0.00005/0.61 = 0.00008$, $Re_1 = 5.8 \times 10^4$ } $f_2 = 0.02$ (see figure on previous slide)

2. New guess D_2 from Darcy-Weisbach equation: $D_2 = \left(\frac{8}{\pi^2 g}\frac{\Delta l Q^2}{h_r}\right)^{1/5} f_2^{1/5} = 0.53$ [m]

3. New guess of Re: $Re_2 = \frac{4Q_2}{\pi D \nu} = 6.7 \times 10^4$

Iteration step 3:

1. New guess of f: $k_s/D = 0.00005/0.53 = 0.0001$, $Re_1 = 6.7 \times 10^4$ } $f_2 = 0.02$ → Solution converged !

Flow regime: smooth turbulent

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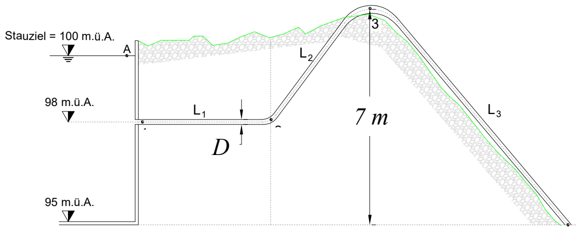
Ex 7. Pipe flow

Siphon: A tube used to convey liquid upwards from a reservoir and then down to a lower level of its own accord. Once the liquid has been forced into the tube, typically by suction or immersion, flow continues unaided



D = 0.5 m
k_s = 0.00015 m
$L_1 = L_2$ = 4 m, L_3 = 8m

Case 1: All minor energy losses are neglected and the fluid considered is water at 20°.

1) Determine the discharge *Q*, the Darcy-Weisbach friction coefficient f, and the flow regime.
2) Draw the total energy line, and the potential energy line. Deduce from both the evolution of the pressure along the pipe.
3) Where does the minimum pressure in the pipe occur and what is its value ?
4) Is there a risk of cavitation in that point ?

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Ex 7. Pipe flow. Solution

1) Determine the discharge *Q*, the Darcy-Weisbach friction coefficient *f*, and the flow regime.

- Energy budget between the free surface in the reservoir and the exit of the siphon:

$$h_1 + \frac{p_1}{\gamma} + \frac{U_1^2}{2g} = h_2 + \frac{p_2}{\gamma} + \frac{U_2^2}{2g} + h_r$$

$$\begin{cases} \Delta h = h_1 - h_2 = 5\text{m} \\ p_1 = p_2 = p_a \\ U_1 = 0 \\ U_2 = U = Q/S \\ h_r = f\dfrac{\Delta l}{D}\dfrac{U^2}{2g} \end{cases}$$

$$\frac{U_2^2}{2g} = \frac{\Delta h}{\left(1 + f\dfrac{\Delta l}{D}\right)}$$

$$Q = \frac{\pi D^2}{4}\left(\frac{2g\Delta h}{1 + f\dfrac{\Delta l}{D}}\right)^{1/2}$$

- The Darcy-Weisbach friction coefficient has to be determined by iteration. Attention: the iterative procedure is slightly different for a pipe system than for a single pipe reach.

Ex 7. Pipe flow. Solution

Solution similar to procedure outlined in application example 2 of TH_Pipeflow

Water at 20°: $\nu = 1 \times 10^{-6}$ [m^2 s^{-1}]

D, h_r, k_s, ν known → Q, U, f, Re to be solved

Iteration step 1:

1. Initial guess of f: $k_s/D = 0.00015/0.5 = 0.0003$; Re is unknown → f_1 = 0.017 (see Ex. 5. Pipe flow).

2. Initial guess of Q from the Darcy-Weisbach equation:

$$Q = \frac{\pi D^2}{4}\left(\frac{2g\Delta h}{1 + f\frac{\Delta l}{D}}\right)^{1/2} = 1.57 \text{ [m}^3\text{ s}^{-1}\text{]}$$

3. Initial guess of Re: $\mathrm{Re}_1 = \frac{4Q_1}{\pi D \nu} = 4.0 \times 10^6$

Iteration step 2:

1. New guess of f: $k_s/D = 0.00015/0.5 = 0.0003$; $\mathrm{Re}_1 = 4.0 \times 10^6$ → f_1 = 0.017 → Solution converged !

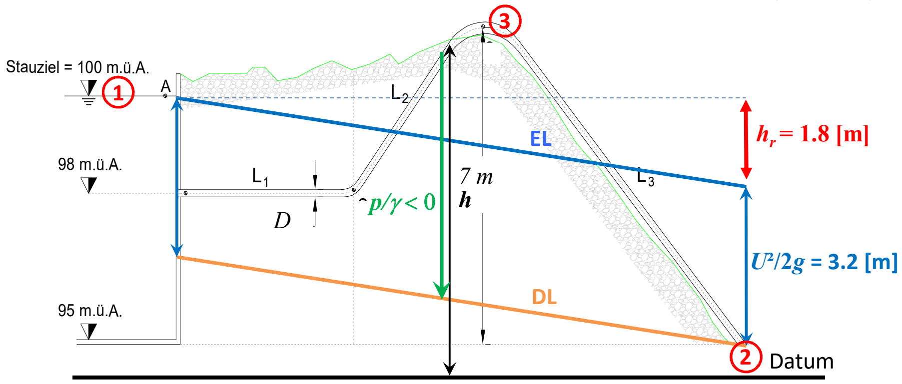
Flow regime very near rough turbulent, implying that f only depends on k_s/D

Ex 7. Pipe flow. Solution

2) Draw the total energy line, and the potential energy line. Deduce from both the evolution of the pressure along the pipe.

EL = Total energy line, $E = h + p/\gamma + U^2/2$

DL = Potential energy line, $p^*/\gamma = h + p/\gamma$



The relative pressure (with respect to the atmospheric pressure) is negative everywhere in the siphon.

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Ex 7. Pipe flow. Solution

3) Where does the minimum pressure in the pipe occur and what is its value ?

The minimum relative pressure occurs in the highest point of the siphon. It is found be making an energy budget between the free surface in the reservoir and this point:

$$h_1 + \frac{p}{\gamma} + \frac{U_1^2}{2g} = h_3 + \frac{p_3}{\gamma} + \frac{U_3^2}{2g} + h_{r,1-3} \longrightarrow \frac{p_3}{\gamma} = h_1 - h_3 - \frac{U^2}{2g}\left(1 + f\frac{\Delta l_{1-3}}{D}\right) = -6.1 \text{ [m]}$$

4) Is there a risk of cavitation in that point ?

The absolute pressure is given by: $\frac{p_{abs}}{\gamma} = \frac{p_a}{\gamma} + \frac{p}{\gamma} = \frac{1.013\text{x}10^5}{9.81\text{x}1000} + \frac{p}{\gamma} \approx 10 + \frac{p}{\gamma} \rightarrow p_{abs,min}$ = 3.9 [m]

Where p_a = 1.013 x 10^5 [N m^{-3}] is the normal atmospheric pressure

As a rule of thumb, the relative pressure p/γ should be larger than -7 [m], or the absolute pressure p_{abs}/γ should be larger than 3 [m]. This value is required because the computed pressure:

- **Does not take into account the 3D distribution of the velocity in the pipe. Higher velocities may locally occur (for example due to 3D effects in the bend), leading to lower pressure.**
- **Does not take into account turbulent fluctuations, which may also lead to higher velocities and lower pressures.**

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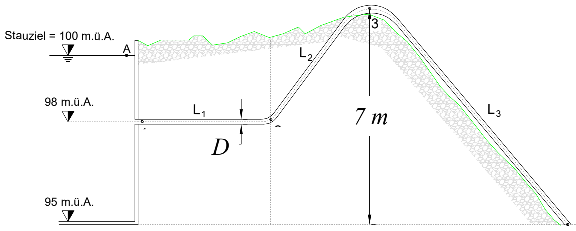
Ex 8. Pipe flow

Siphon: A tube used to convey liquid upwards from a reservoir and then down to a lower level of its own accord. Once the liquid has been forced into the tube, typically by suction or immersion, flow continues unaided



D = 0.5 m
k_s = 0.00015 m
$L_1 = L_2$ = 4 m, L_3 = 8m

Case 2: Minor energy losses occur at the pipe inflow (K = 0.2), and in the two bends (K = 0.3 for each bend). The fluid considered is water at 20°.

1) Determine the discharge *Q*, the Darcy-Weisbach friction coefficient f, and the flow regime.
2) Draw the total energy line, and the potential energy line. Deduce from both the evolution of the pressure along the pipe.
3) Where does the minimum pressure in the pipe occur and what is its value ?
4) Is there a risk of cavitation in that point ?

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Ex 8. Pipe flow. Solution

1) Determine the discharge *Q*, the Darcy-Weisbach friction coefficient *f*, and the flow regime.

- Energy budget between the free surface in the reservoir and the exit of the siphon:

$$h_1 + \frac{p_1}{\gamma} + \frac{U_1^2}{2g} = h_2 + \frac{p_2}{\gamma} + \frac{U_2^2}{2g} + h_r + \sum h_m$$

$$\frac{U^2}{2g} = \frac{\Delta h}{1 + f\frac{\Delta l}{D} + K_{\text{inflow}} + K_{\text{bend,1}} + K_{\text{bend,2}}}$$

$$Q = \frac{\pi D^2}{4}\left(\frac{2g\Delta h}{1 + f\frac{\Delta l}{D} + K_{\text{inflow}} + K_{\text{bend,1}} + K_{\text{bend,2}}}\right)^{1/2}$$

$$\begin{cases} \Delta h = h_1 - h_2 = 5\text{m} \\ p_1 = p_2 = p_a \\ U_1 = 0 \\ U_2 = U = Q/S \\ h_r = f\frac{\Delta l}{D}\frac{U^2}{2g} \\ \sum h_m = \left(K_{\text{inflow}} + K_{\text{bend,1}} + K_{\text{bend,2}}\right)\frac{U^2}{2g} \end{cases}$$

- The Darcy-Weisbach friction coefficient has to be determined by iteration. Attention: the iterative procedure is slightly different for a pipe system than for a single pipe reach.

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Ex 8. Pipe flow. Solution

Solution similar to procedure outlined in application example 2 of TH_Pipeflow

Water at 20°: $\nu = 1\text{x}10^{-6}$ [$m^2\ s^{-1}$]

D, h_r, k_s, ν known → Q, U, f, Re to be solved

Iteration step 1:

1. Initial guess of f: $k_s/D = 0.00015/0.5 = 0.0003$; Re is unknown → f_1 = 0.017 (see Ex. 5. Pipe flow).
2. Initial guess of Q from the Darcy-Weisbach equation:

$$Q = \frac{\pi D^2}{4}\left(\frac{2g\Delta h}{1 + f\frac{\Delta l}{D} + K_{\text{inflow}} + K_{\text{bend,1}} + K_{\text{bend,2}}}\right)^{1/2} = 1.27\ [\text{m}^3\ \text{s}^{-1}]$$

3. Initial guess of Re: $\text{Re}_1 = \frac{4Q_1}{\pi D \nu} = 3.2\ \text{x}\ 10^6$

Iteration step 2:

1. New guess of f: $k_s/D = 0.00015/0.5 = 0.0003$; $\text{Re}_1 = 3.2\ \text{x}\ 10^6$ → f_1 = 0.017 → Solution converged !

Flow regime very near rough turbulent, implying that f only depends on k_s/D

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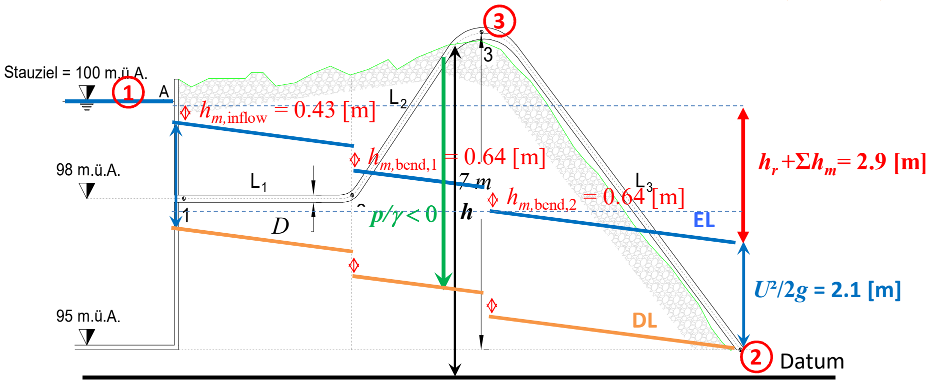
Ex 8. Pipe flow. Solution

2) Draw the total energy line, and the potential energy line. Deduce from both the evolution of the pressure along the pipe.

EL = Total energy line, $E = h + p/\gamma + U^2/2$

DL = Potential energy line, $p^*/\gamma = h + p/\gamma$



The relative pressure (with respect to the atmospheric pressure) is negative everywhere in the siphon.

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Ex 8. Pipe flow. Solution

3) Where does the minimum pressure in the pipe occur and what is its value ?

The minimum relative pressure occurs in the highest point of the siphon. It is found be making an energy budget between the free surface in the reservoir and this point:

$$h_1 + \frac{p_1}{\gamma} + \frac{U_1^2}{2g} = h_3 + \frac{p_3}{\gamma} + \frac{U^2}{2g} + h_{r,1-3} + \left(K_{\text{inflow}} + K_{\text{bend,1}} + K_{\text{bend,2}}\right)\frac{U^2}{2g}$$

$$\rightarrow \quad \frac{p_3}{\gamma} = h_1 - h_3 - \frac{U^2}{2g}\left(1 + f\,\frac{\Delta l_{1-3}}{D} + K_{\text{inflow}} + K_{\text{bend,1}} + K_{\text{bend,2}}\right) \quad = -6.4 \text{ [m]}$$

Note that the most conservative estimation of the minimum pressure is obtained by taking also into account the minor energy losses in the second bend

4) Is there a risk of cavitation in that point ?

The absolute pressure is given by: $\frac{p_{abs}}{\gamma} = \frac{p_a}{\gamma} + \frac{p}{\gamma} = \frac{1.013\text{x}10^5}{9.81\text{x}1000} + \frac{p}{\gamma} \approx 10 + \frac{p}{\gamma}$ → $p_{abs,min}$ = 3.6 [m]

Where p_a = 1.013 x 10^5 [N m^{-3}] is the normal atmospheric pressure

Taking into account the minor energy losses slightly increases the cavitation risk in this particular case. The rule of thumb that p/γ should be larger that -7 [m] is still marginally satisfied, and the risk of cavitation cannot be neglected.