VORLESUNG

TECHNISCHE HYDRAULIK

222.564

Exercises

Euler momentum theorem – Bernoulli‘s equation – Open-channel flow

Ex 1. Euler momentum theorem

Opponitz Kraftwerk , Wien Energie, Austria

Input data:

- $\rho = 1000$ [kg m^{-3}]
- $\mu = 1.0 \times 10^{-3}$ [kg m^{-1} s^{-1}]
- $Q = 8$ [m^3 s^{-1}]
- $D = 1$ [m]
- $\Delta h = 115$ [m]
- $\Delta l = 160$ [m]
- Penstock in galvanized steel: $k_s = 0.15 \times 10^{-3}$ [m]

Let us consider again the penstock pipe of the Opponitz power plant. Let us consider again the case detailed in TH_PipeFlow, where a discharge of Q = 8 [m^3 s^{-1}] is obtained by partially closing a valve that is situated at the downstream end of the penstock.

Imagine now that the penstock pipe is not straight, but has a change in direction of 75 [°] near its downstream end. Assume that the last 20 [m] of the penstock, including the bend and the valve, are flat.

Determine the force induced by the flow on the pipe due to this change in direction. Determine the total force, the downslope component of the force, and the transverse component of the force for two configurations: the first with the bend just upstream of the valve and the second with the bend just downstream of the valve. For what configuration is the force smallest ?

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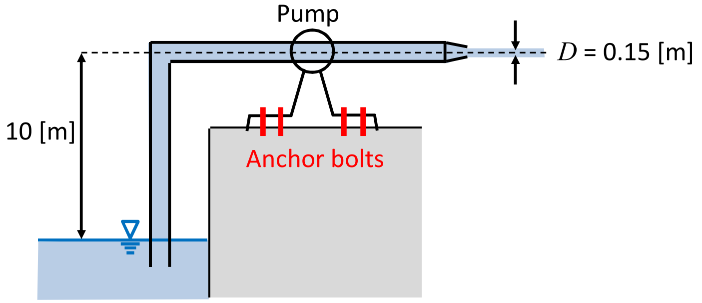
We have considered this example already in TH_PipeFlow

Ex 2. Euler momentum theorem

A horizontal jet is generated by pumping water out of a large reservoir. The discharge of the jet is Q = 0.1 [m^3 s^{-1}] and the jet diameter is D = 0.15 [m]. The free surface of the reservoir is situated 10 [m] below the jet axis. Energy losses in the system can be neglected.



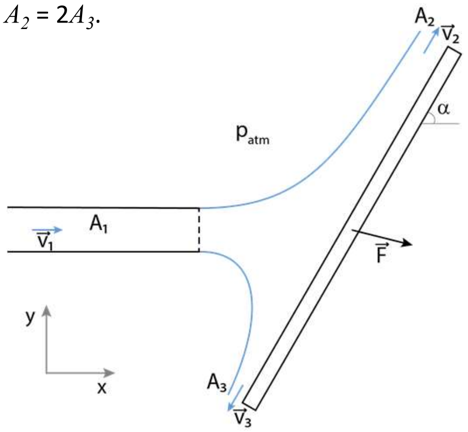
1. Determine the energy head introduced by the pump in the system.
2. How many bolts are required to anchor the pump, if one bolt has an admissible shear force of 25 [N] ?

Ex 2. Euler momentum theorem

Water (ρ = 1000 [kg m^{-3}]) is pumped out of a pipe with cross-sectional area A_1 = 0.2 [m^2] at a velocity of V_1 = 10 [m s^{-1}] before it hits a stationary plate that is tilted at an angle α = 60 [°] relative to the incoming jet. The incoming jet is split into two outgoing jets. Assume that the effects of gravity can be neglected. The jet is surrounded by air at atmospheric pressure. The figure pictures the top view of the jet exiting the pipe and hitting the plate.

1. Prove that the magnitudes of the outgoing velocities V_2 and V_3 have to be equal to the magnitude of the velocity V_1 of the incoming jet using Bernoulli's theorem.

2. Choose a control volume and calculate the x- and y-component of the total force F the jet exerts on the plate. Assume that A_2 = 2A_3.



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Ex 1. Bernoulli's equation

We have discussed in TH_Introduction the very original and surprising hunting strategy of the pistol shrimp. By quickly closing its clamp, the pistol shrimp generates a high velocity jet, which leads to the generation of a cavitation bubble. The pressure shock generated by the subsequent explosion of the cavitation bubble kills the pray.

Imagine that the pistol shrimp is at the bottom of a 0.5 [m] deep aquarium filled with water at 20 [°]. Estimate the velocity of the jet that the pistol shrimp creates.

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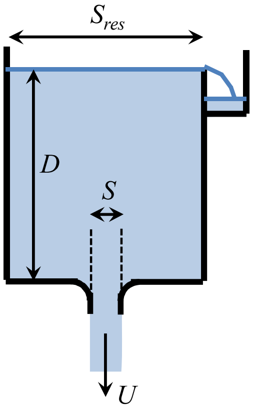
Ex 2. Bernoulli's equation. Torricelli's formula (1644)



Consider a reservoir of constant surface area S_{res} filled with a fluid of density ρ [kg m^{-3}]. An overflow is used to maintain the fluid at a constant level D [m] in the reservoir. The fluid flows out of the reservoir in the form of a jet through an opening in the bottom of surface area S [m^2].

1. Determine the velocity U [m s^{-1}] in the jet.
2. Determine the discharge flowing out through the bottom opening.
3. Consider now the emptying of the reservoir. Assume that $S << S_{res}$, such that the variation of the free surface in the reservoir is very slow, and the problem can be considered as quasi-stead. Determine the time required to empty the reservoir.

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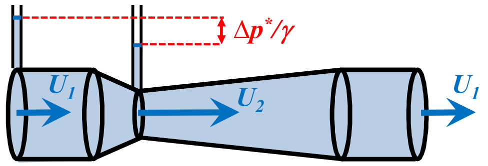
Ex 3. Bernoulli's equation. The Venturi tube (~1800)

A Venturi tube is a pipe that consists of a contraction followed by an expansion (Figure). Demonstrate that velocity can be derived from the pressure difference between the upstream section and the contracted section.

Note that energy losses are typically negligible in converging flow, whereas they can be substantial in diverging flows, especially when flow separates from the walls and recirculation zones form. For that reason, the contraction in Venturi tubes is typically rather abrupt, whereas the divergence is typically more gradual.

Venturi tube



Venturi tube on the hull of an airplane for velocity measurements

Ex 1. Open-channel flow

Consider a reach on the Danube near Vienna.

1. Schematize the geometry of the river system and justify your schematization.

   In practice, it is important to treat problems with the appropriate level of complexity. For example, what level of complexity do you retain in the schematization of the river shape ? Can the cross-sectional shape be approximated by a trapezium ? Or is the effect of the banks negligible and can it be approximated by a rectangle, which simplifies calculations. Can the bottom slope be taken as constant in the considered reach ?

2. Choose a discharge Q (for example the mean annual discharge).
3. Draw the specific energy curves for Q.
4. Compute the critical flow depth (D_c) and the corresponding specific energy ($E_{s,c}$) for this Q and indicate them on the specific energy curve.
5. Make an estimation of the friction factor and justify your estimation.
6. Compute the normal flow depth (D_n) for Q, and represent it on the specific energy curves.
7. Define the flow regime.
8. Due to construction works, the width has to be reduced by 50m over a length of 500m. Based on specific energy considerations, compute the local variation in the elevation of the water surface resulting from this width reduction.

Solve the exercise for 2 cases: a subcritical river and a supercritical one

Ex 2. Open-channel flow

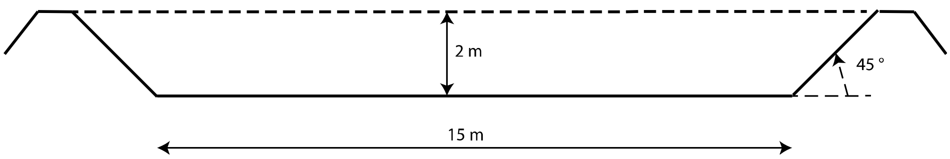
Consider the same reach on the Danube as in Ex. 1 and the same discharge. Assume that the reach has a constant geometry and is sufficiently long for normal flow conditions to establish. A sluice gate is installed over the entire width that locally reduces the flow depth to 0.5 [m].

1. Draw schematically the backwater curves upstream and downstream of the sluice gate. Indicate in your scheme the normal and critical flow depths, and name the types of backwater curve that occur.
2. Compute the backwater curves upstream and downstream of the sluice gate.
3. If a hydraulic jump occurs, determine the conjugate depths and determine its location (distance from the sluice gate). The length of the hydraulic jump can be neglected.

Ex 3. Open-channel flow

Consider a mountain river with the following characteristics:

- A longitudinal bottom slope of 0.02.
- A trapezoidal cross-section with bottom width of 15 [m], banks inclined at 45[°], and bank height of 2 [m] (Figure).
- A roughness coefficient according to Manning-Strickler of K_s = 30 [$m^{1/3}$ s^{-1}].



1. Compute the hydraulic capacity of the river, which is also called the bankfull discharge. Assume that flow is normal.
2. Compute the normal and critical flow depths for the bankfull discharge.
3. Identify the flow regime.
4. Draw the specific energy curve for the bankfull discharge and indicate the normal and critical flows. Consider a depth range of 0 to 6 m for drawing the curve.
5. Compute the bed shear stress for the bankfull discharge.

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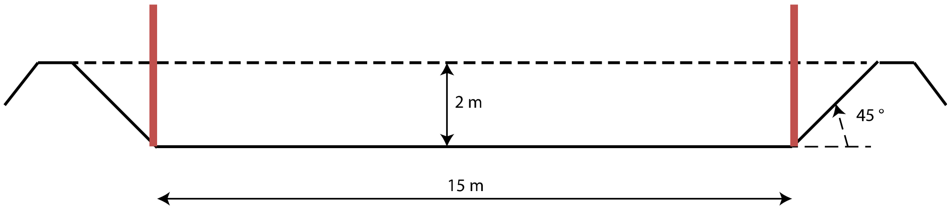
Ex 4. Open-channel flow

Consider the same mountain river as in Ex. 3. Due to the crossing of a highway bridge, the banks are locally vertical, but the bottom width is maintained at 15 m, leading to a local constriction of the flow (Figure).



1. Draw the specific energy curve in the constricted reach for the bankfull discharge identified in Ex. 3. Superpose this on the specific energy curve drawn in Ex. 3.
2. By how much do the banks have to be raised in order to maintain the hydraulic capacity, i.e. in order to avoid inundations.
3. A hydraulic jump will occur upstream of the constriction. Compute the conjugate flow depths (i.e. flow depths just upstream and downstream of the hydraulic jump) and compute the energy losses in the hydraulic jump.
4. Draw schematically the longitudinal profiles of the bed, water surface and energy line in the reach upstream of the constriction; indicate also the normal and critical flow depths.
5. Illustrate the evolution of the water depth on the specific energy curve.

Ex 5. Open-channel flow

We have seen in TH_OpenChannelFlow_1 that discharge can be measured by imposing critical flow conditions. We have treated the example of imposing critical flow by means of a bottom step (the relevant slide is reproduced in the figure below). Develop explicitly the relation $Q = Q\,(D_{upstream})$.

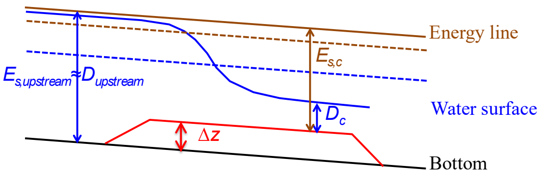
Discharge measurements based on relation *Q*-D_c

Critical flow:
$D_c = D_c(Q)$ and $Q = Q(D_c)$

$E_{s,c} = E_{s,c}(Q)$ and $Q = Q(E_{s,c})$

$E_{s,c} \approx E_{upstream} - \Delta z \approx D_{upstream} - \Delta z$

$\rightarrow Q = Q(D_{upstream})$



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Exercise from the lecture TH_Hydrostatics

Ex 6. Open-channel flow

A discharge of Q = 12 [m^3 s^{-1}] is flowing in a 2 [m] wide rectangular channel. The Manning-Strickler roughness coefficient is estimated at K_s = 40 [m$^{1/3}$ s^{-1}]. The flow regime will obviously depend on the channel slope. For a mild slope the flow will tend to be subcritical, whereas it will tend to be supercritical for a steep slope. Determine the critical slope, i.e. the one that differentiates between mild-slope (*M*-type backwater curves) and steep-slope (S-type backwater curves) characterizations of the channel.

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Exercise from the lecture TH_Hydrostatics