Technische Universität Wien
Institut für Automatisierungs- und Regelungstechnik

SCHRIFTLICHE PRÜFUNG zur
VU Modellbildung am 29.11.2013

LÖSUNG

Aufgabe 1:

a)

$$m_k \ddot{s} = A p_g - A p_o$$

b)

$$\begin{aligned} V_o &= A(l-d-s) \\ \dot{m}_o &= \frac{\partial}{\partial t}(\rho_o V_o) = \dot{\rho}_o V_o + \rho_o \dot{V}_o \\ \rho_o q &= \frac{1}{\beta}\rho_o \dot{p}_o + \rho_o \dot{V}_o \end{aligned}$$

$$\dot{p}_o = \frac{1}{V_o}\beta\,(q + A\dot{s})$$

c)

$$\begin{aligned} V_g &= As \\ 0 &= \dot{m}_g = \frac{\partial}{\partial t}(\rho_g V_g) = \dot{\rho}_g V_g + \rho_g \dot{V}_g \\ 0 &= \frac{\rho_g \dot{p}_g}{\kappa p_g} V_g + \rho_g \dot{V}_g \end{aligned}$$

$$\dot{p}_g = -\frac{\kappa}{V_g} p_g A \dot{s}$$

d)

$$\dot{E}_i = \dot{Q}$$

$$\dot{T}_k = \frac{1}{m_k c_p}\left(-\alpha_{g,k} A (T_k - T_g) - \alpha_{o,k} A (T_k - T_o)\right)$$

Aufgabe 2:

a) Siehe Abbildung 1.

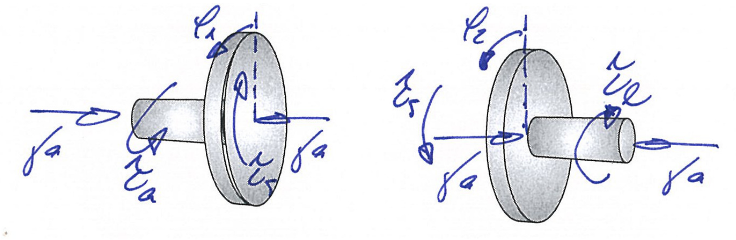

Abbildung 1: Skizze zum Freischneiden der Reibkupplung.

b)

$$\theta_1 \ddot{\varphi}_1 = -\tau_r + \tau_a$$
$$\theta_2 \ddot{\varphi}_2 = \tau_r - \tau_l$$

$$\varphi_1(t) = -\frac{1}{2}\frac{\tau_r - \tau_a}{\theta_1}t^2 + \omega_{1,0}t + \varphi_{1,0}$$
$$\varphi_2(t) = \frac{1}{2}\frac{\tau_r - \tau_l}{\theta_2}t^2 + \omega_{2,0}t + \varphi_{2,0}$$

c) Kuppelzeit t_k aus $\omega_1(t_k) = \omega_2(t_k)$

$$t_k = \frac{\omega_{1,0} - \omega_{2,0}}{\frac{\tau_r - \tau_a}{\theta_1} + \frac{\tau_r - \tau_l}{\theta_2}}$$

d)

$$\tau_r = \frac{2\mu f_a (r_a^3 - r_i^3)}{3(r_a^2 - r_i^2)}$$

Aufgabe 3:

a)

$$\mathbf{r}_s = \begin{bmatrix} s + l_S \sin(\varphi) \\ -l_S \cos(\varphi) \\ 0 \end{bmatrix}, \quad \dot{\mathbf{r}}_s = \begin{bmatrix} v + l_S \cos(\varphi)\omega \\ l_S \sin(\varphi)\omega \\ 0 \end{bmatrix}$$

b)

$$T = \frac{1}{2}m_W v^2 + \frac{1}{2}m_S l_S^2 \omega^2 + m_S v l_S \cos(\varphi)\omega + \frac{1}{2}m_S v^2 + \frac{1}{2}\theta_{S,zz}^{(S)}\omega^2$$

c)

$$V_f = \frac{1}{2}c_W (s - s_{W0})^2$$

d)

$$V_g = l_S m_S g(1 - \cos(\varphi))$$

e)

$$\begin{aligned} L = &\frac{1}{2}\left(m_S l_S^2 + \theta_{S,zz}^{(S)}\right)\omega^2 + m_S v l_S \cos(\varphi)\omega - \frac{1}{2}c_W s^2 + c_W s_{W0} s \\ &+ \frac{1}{2}v^2(m_S + m_W) + m_S g l_S \cos(\varphi) - \frac{1}{2}c_W s_{W0}^2 - m_S g l_S \end{aligned}$$

f)

$$\begin{aligned} (m_W + m_S)\,\ddot{s} + m_S l_S \cos(\varphi)\,\ddot{\varphi} - m_S l_S \sin(\varphi)\,\dot{\varphi}^2 + c_W\,(s - s_{W0}) &= f_e - d_R \dot{s} \\ m_S l_S \cos(\varphi)\,\ddot{s} + \left(\theta_{S,zz}^{(S)} + m_S l_S^2\right)\ddot{\varphi} + m_S g l_S \sin(\varphi) &= 0 \end{aligned}$$

Aufgabe 4:

a)

$$y_s = -\frac{4r}{3\pi}, \quad x_s = 0$$

b)

$$\begin{aligned} J_k &= \frac{1}{3}\left(\int_{\mathcal{V}}(y^2 + z^2)\mathrm{d}m + \int_{\mathcal{V}}(x^2 + z^2)\mathrm{d}m + \int_{\mathcal{V}}(x^2 + y^2)\mathrm{d}m\right) \\ &= \frac{2}{3}\int_{\mathcal{V}} r^2 \mathrm{d}m = \frac{8}{15}\rho\pi R^5 \end{aligned}$$