Computation exercise 2: Control design

Mechatronic systems
376.050
2013W

Important: Answers must be a hard copy and submitted to the office in CA0421 by January 8, 2014 at 4pm. The work must be original.

The position of a laser pickup in a CD/DVD player is controlled, such that it focuses its laser spot on a disk. This positioning system can be modeled as shown in Fig. 1, where mass *m* is 10^{-3} kg, spring constant *k* is 40 N/m and damping coefficient *c* is 0.05 N/(m/s). Fig. 2 shows a control block diagram, where *P(s)* is a transfer function from the actuation force *F* to the position *x*. A feedback controller is described as *C(s)*, and *d* is disturbance.

Design two types of PID controllers, answering questions below.

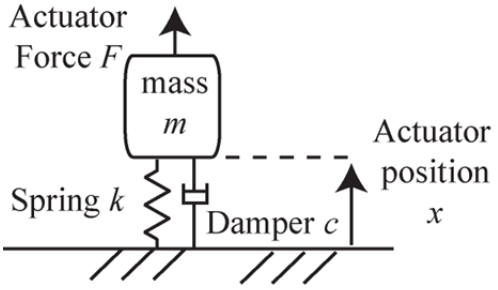

Fig. 1: A lumped mass model of a laser pickup.

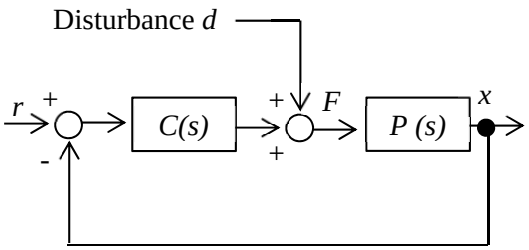

Fig. 2: Control block diagram.

a. Derive transfer function *P(s)*. [5%]

b. A PID controller cascaded with a first-order low-pass filter has 2 poles and 2 zeros and can be described with PID gains, as well as the cut-off frequency ω_{lpf}.

$$C(s) = \frac{k_d s^2 + k_p s + k_i}{s\left(\frac{s}{\omega_{lpf}} + 1\right)},$$

Design a PID controller with a first-order low-pass filter, fulfilling the following conditions (rule of thumb in the text book). Also simulate a Bode plot of the open-loop transfer function *C(s)P(s)* for validation. [25%]

- The open-loop cross-over frequency ω_c is $2\pi\times10^3$ rad/s (1 kHz). (i.e. $|C(s)P(s)|$ is 0 dB at 1 kHz).
- The integral action terminates at $0.1\omega_c$.
- The derivative action starts at $0.33\omega_c$.
- The cutoff frequency of the low-pass filter is given at $3.3\omega_c$ (i.e. $\omega_{lpf} = 3.3\omega_c$).

c. Design a PID controller with a first-order low-pass filter, fulfilling the following conditions (pole-zero cancellation control). Also simulate a Bode plot of the open-loop transfer function *C(s)P(s)* for validation. [25%]

- The open-loop cross-over frequency ω_c is $2\pi\times10^3$ rad/s (1 kHz).
- The poles of P(s) are cancelled by the controller zeros.
- The cutoff frequency of the low-pass filter is given at $3.3\omega_c$.

d. Simulate Bode plots of *x(s)/d(s)* and *x(s)/r(s)* for the closed-loop system with the controller designed in (b) and also (c). [15%]

e. Simulate step response with *r* as the input and *x* as the output for the closed-loop system with the controller designed in (b) and also (c). Also draw step response with *d* as the input and *x* as the output for the system. [15%]

f. Using the results of the above simulations, discuss which controller is more suitable for this application. [15%]