Computation exercise 3(b): Control design

<div align="right">Mechatronic systems
376.050
2014W</div>

Important: Answers must be a hard copy and submitted to the office in CA0421 by January 20, 2015 at 4pm. The work must be original.

Fig. 1 shows a lumped mass model of a positioning system using a piezo actuator, where *F* is the input and *x* is the output. Fig. 2 shows a control block diagram to regulate the position *x*, where *P(s)* is the plant. The transfer function *C(s)* is a controller and has the following structure:

$$C(s) = C_I(s)C_{notch}(s),$$

where C_I is an I controller and C_{notch} is a notch filter. Following the questions below, design and implement a feedback controller *C(s)*.

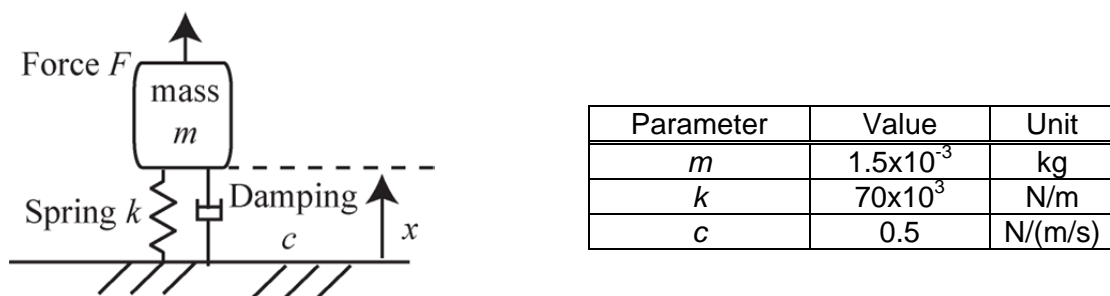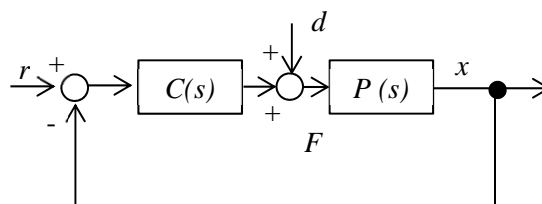| Parameter	Value	Unit
m	1.5×10^{-3}	kg
k	70×10^3	N/m
c	0.5	N/(m/s)

Fig. 1: A lumped mass model of a positioning system.

Fig. 2: Control block diagram.

i. Derive transfer function *P(s)* and simulate a Bode plot. [5%]

ii. Design the notch filter, such that the peak in the magnitude of *P(s)* is trimmed in the Bode plot of (i). For validation, add a simulated Bode plot of the transfer function $C_{notch}(s)P(s)$ on the figure of (i). [15%]

iii. Design $C_I(s)$ to regulate the plant cascaded with the notch filter (i.e. $C_{notch}(s)P(s)$), fulfilling the following conditions. Also simulate a Bode plot of the open-loop transfer function $C_I(s)C_{notch}(s)P(s)$ for validation. [20%]

   - Phase margin should be 40 deg or more.
   - Gain margin should be 10 dB or more.
   - The open-loop cross-over frequency should be as high as possible.

iv. Simulate step response of the closed-loop system with *r* as the input and *x* as the output. Also draw step response with the disturbance *d* as the input and *x* as the output. Using the results, discuss the influence of the notch filter on these inputs. [20%]

v. To implement *C(s)* designed in (iii) as a digital filter on a microcontroller running at a sampling frequency of 10 kHz, discretize *C(s)*. Special care should be taken not to change the notch

frequency and the cross over frequency. For validation, show Bode plots of the continuous and the discrete controller (i.e. *C(s)* and *C(z)*) on one figure. [20 %]

vi.   To implement *C(z)* with gains and memories (time delays), draw a block diagram showing an IIR filter structure and derive the coefficients of the gains. [20 %]