Lösungen zur 1. Teilprüfung

Signale und Systeme 2, Sommersemester 2008

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24.4.2008 C. Novak

Gruppe A

- **Beispiel 1**

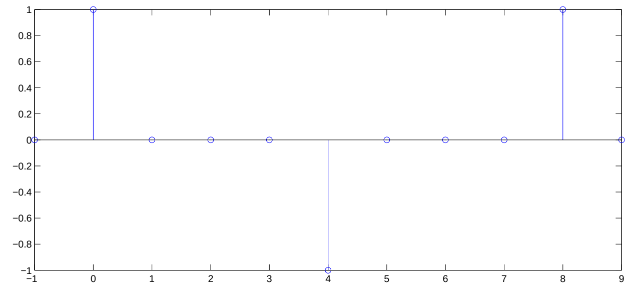
  a) $N_1 = 8$

  b) $x_1[n]$ ist gerade

  c) $x_1[n] = cos\frac{\pi}{4}n \rightarrow c_k = \frac{1}{2}\delta[k-1] + \frac{1}{2}\delta[k+1]$

  d) $N_2 = 4$

  e) Skizze (Signal wird periodisch fortgesetzt)

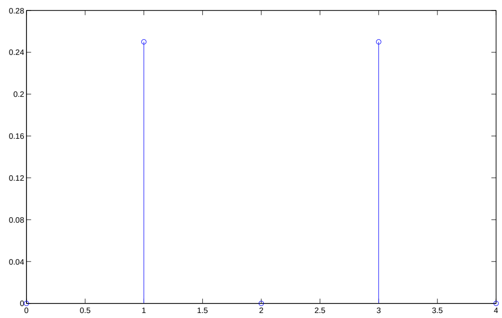


  $N = 8$

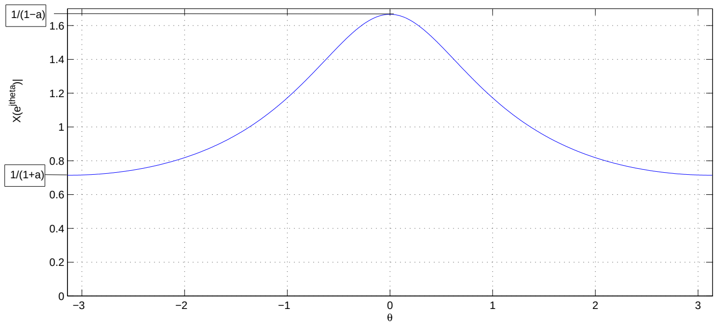
  f) $c_k = \frac{1}{8}\left(1 - (-1)^k\right)$

  f1,2) c_k sind reell und gerade ($x[n] = x[-n]$)

  f3) Skizze (Koeffizienten werden periodisch fortgesetzt)



- **Beispiel 2**
  a) Wertebereich: $a \in [-1; 1]$
  b) $x[n] = a^n \sigma[n] \rightarrow X(e^{(j\theta)} = \frac{1}{1-ae^{(-j\theta)}}$ Skizze



  d) $\tilde{x}[n] = a^n \sigma[n] - a^n \sigma[n-(N+1)] = a^n \sigma[n] - a^{N+1} a^{n-N-1} \sigma[n-N-1]$
  $\rightarrow X(e^{(j\theta)} = \frac{1}{1-ae^{(-j\theta)}} \left(1 - a^{N+1} e^{-j\theta(N+1)}\right)$

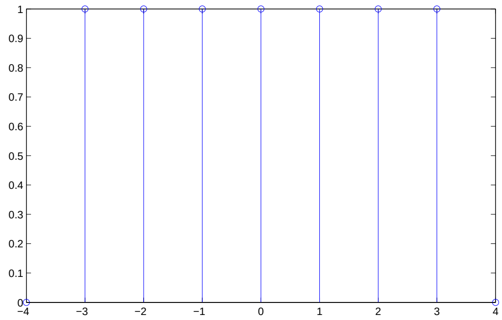
  e) $P_x = \sum_{n=0}^{\infty} |x[n]|^2 = \sum_{n=0}^{\infty} (a^2)^n = \frac{1}{1-a^2}$
  $P_{\tilde{x}} = \sum_{n=0}^{N} (a^2)^n = \frac{1-a^{2(N+1)}}{1-a^2}$
  $\alpha = 1 - a^{2(N+1)}$

- **Beispiel 3**

  a) System ist akausal ($h[n] \neq 0$ für $n < 0$)

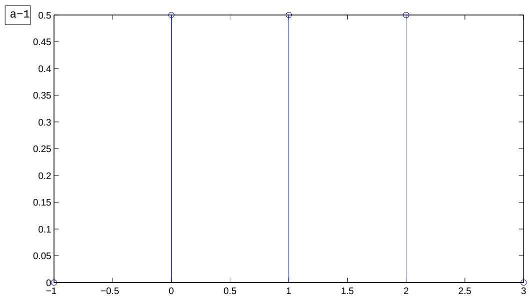
  b) Zerlegung in 2 Teilsysteme: $h[n] = h_1[n] + h_2[n]$

  Skizze von $h_1[n]$



  $\rightarrow H_1(e^{j\theta}) = \frac{\sin(7\theta/2)}{\sin(\theta/2)}$

  Skizze von $h_2[n]$



  $H_2(e^{j\theta}) = (a-1)e^{-j\theta}\frac{\sin(3\theta/2)}{\sin(\theta/2)}$

  $H(e^{j\theta}) = H_1(e^{j\theta}) + H_2(e^{j\theta}) = \frac{\sin(7\theta/2)}{\sin(\theta/2)} + (a-1)e^{-j\theta}\frac{\sin(3\theta/2)}{\sin(\theta/2)}$

  c) 2 Lösungswege:
  i) $H(e^{j\theta}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\theta} \rightarrow H(e^{j\theta})|_{\theta=0} = \sum_{n=-\infty}^{\infty} x[n] = 4 + 3a$
  ii) mittels Ergebnis aus Punkt b) $H(e^{j\theta})|_{\theta=0} = 7 + 3(a-1) = 0$
  $\rightarrow a = -4/3$

  d) $y[n] = 0, \quad \forall n$ (Filter sperrt Gleichanteil!)

Gruppe B

- **Beispiel 1**

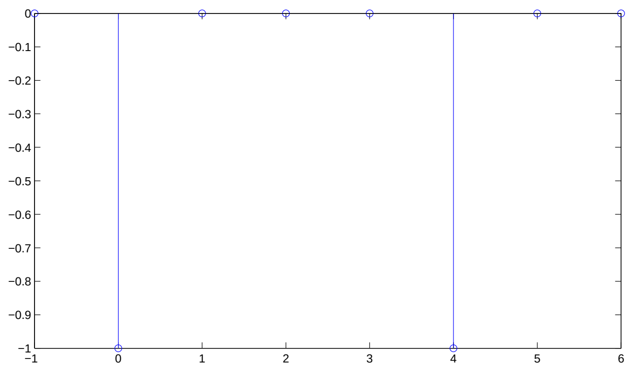
  a) $N_1 = 8$

  b) $x_1[n]$ ist gerade

  c) $x_1[n] = -cos\frac{\pi}{4}n \rightarrow c_k = -\frac{1}{2}\delta[k-1] - \frac{1}{2}\delta[k+1]$

  d) $N_2 = 8$

  e) Skizze (Signal wird periodisch fortgesetzt)



  $N = 4$

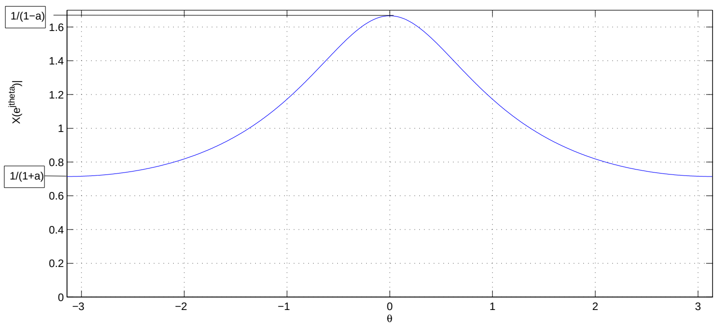
  f) $c_k = -\frac{1}{4}$

  f1,2) c_k sind reell und gerade ($x[n] = x[-n]$)

- **Beispiel 2**

  a) Wertebereich: $a \in [-1; 1]$

  b) $x[n] = -a^n \sigma[n] \rightarrow X(e^{(j\theta)} = -\frac{1}{1-ae^{(-j\theta)}}$ Skizze



  d) $\tilde{x}[n] = -a^n \sigma[n] - (-a^n \sigma[n-(N+1)]) = a^n \sigma[n] + a^{N+1} a^{n-N-1} \sigma[n - N - 1]$
  $\rightarrow X(e^{(j\theta)} = \frac{1}{1-ae^{(-j\theta)}} \left(a^{N+1} e^{-j\theta(N+1)} - 1\right)$

  e) $P_x = \sum_{n=0}^{\infty} |x[n]|^2 = \sum_{n=0}^{\infty} (a^2)^n = \frac{1}{1-a^2}$
  $P_{\tilde{x}} = \sum_{n=0}^{N} (a^2)^n = \frac{1-a^{2(N+1)}}{1-a^2}$
  $\alpha = 1 - a^{2(N+1)}$

- **Beispiel 3**

  a) System ist akausal ($h[n] \neq 0$ für $n < 0$)

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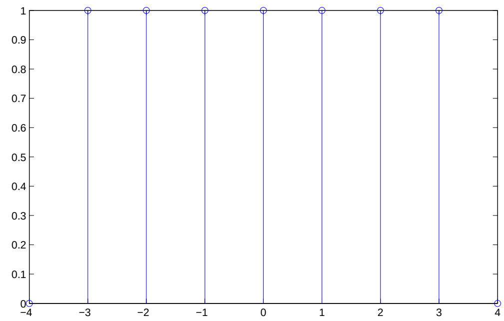
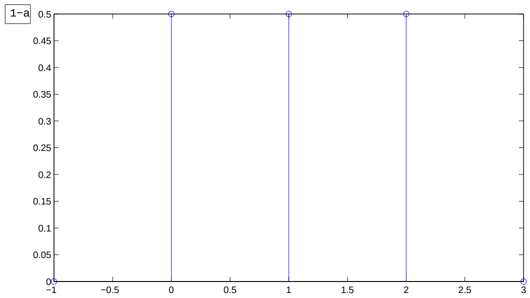
  Skizze von $h_1[n]$



  $\rightarrow H_1(e^{j\theta}) = \frac{\sin(7\theta/2)}{\sin(\theta/2)}$

  Skizze von $h_2[n]$



  $H_2(e^{j\theta}) = (1-a)e^{-j\theta}\frac{\sin(3\theta/2)}{\sin(\theta/2)}$

  $H(e^{j\theta}) = H_1(e^{j\theta}) - H_2(e^{j\theta}) = \frac{\sin(7\theta/2)}{\sin(\theta/2)} - (1-a)e^{-j\theta}\frac{\sin(3\theta/2)}{\sin(\theta/2)}$

  c) 2 Lösungswege:

  i) $H(e^{j\theta}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\theta} \rightarrow H(e^{j\theta})|_{\theta=0} = \sum_{n=-\infty}^{\infty} x[n] = 4 + 3a$

  ii) mittels Ergebnis aus Punkt b) $H(e^{j\theta})|_{\theta=0} = 7 - 3(1-a) = 0$

  $\rightarrow a = -4/3$

  d) $y[n] = 0, \quad \forall n$ (Filter sperrt Gleichanteil!)