Lösungen zur 2. Teilprüfung
Signale und Systeme 2, Sommersemester 2008

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18.6.2008 C. Novak

Gruppe A

- **Beispiel 1**

  a) $y[n] = x[n] + \alpha y[n-1]$

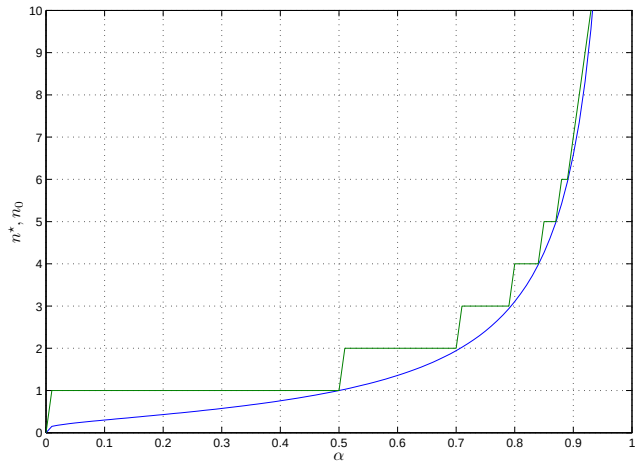
  b) $H(z) = \frac{z}{z-\alpha}$

  c1) $y[n] = \frac{1}{1-\alpha}(1-\alpha^{n+1})\sigma[n]$

  c2) $\lim\limits_{n\to\infty} y[n] = \frac{1}{1-\alpha}$

  c3) $n_0 = \frac{\log(1/2)}{\log\alpha} - 1$ Rundungsoperation auf nächste ganze Zahl: $n^\star = \lceil n_0 \rceil$

  c4) Skizze:

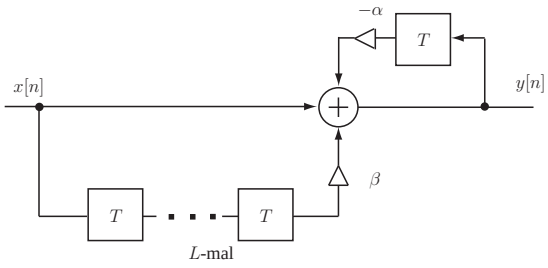


- **Beispiel 2**

  a) $H(z) = \frac{1+\beta z^{-L}}{1+\alpha z^{-1}}$

  b) Pole: $-\alpha, 0(L-1)$ fach; Nullstellen: $z^L = -\frac{1}{\beta} \rightarrow z = |\beta|^{1/L} e^{j(\arg(\beta)+2\pi k)/L}$

  c) Schaltbild:



  d) $h[n] = (-\alpha)^n \sigma[n] + \beta(-\alpha)^{n-L}\sigma[n-L]$

  e) Achtung: $h[n]$ ist unendlich lang, daher ist Abtastung von $H(z)$ falsch! (zeitliches Aliasing)

  Fallunterscheidung: a) $L > N$: $H_a[k] = \sum_{n=0}^{N-1} (-\alpha)^n e^{-j\frac{2\pi}{N}nk} = \frac{1-(-\alpha)^N}{1+\alpha e^{-j\frac{2\pi}{N}k}}$

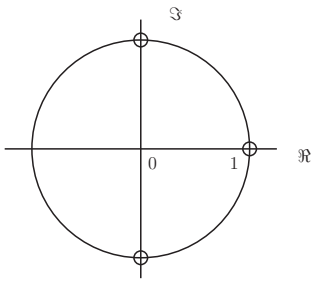
  b) $L < N$: $H_b[k] = H_a[k] + \sum_{n=0}^{N-1} \beta(-\alpha)^{n-L}\sigma[n-L]e^{-j\frac{2\pi}{N}nk}$

  $H_b[k] = H_a[k] + \beta \sum_{n=0}^{N-1-L} (-\alpha)^n \sigma[n-L] e^{-j\frac{2\pi}{N}k(n+L)}$

  $H_b[k] = H_a[k] + \beta \frac{e^{-j\frac{2\pi}{N}kl} + \alpha}{1+\alpha e^{-j\frac{2\pi}{N}k}}$

- **Beispiel 3**

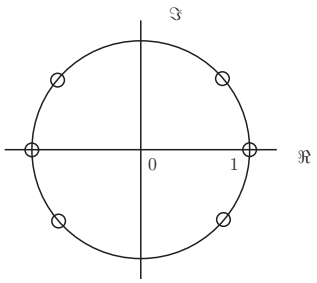
  a) Pol: ∞ (3-fach), Nullstellen: $1, j, -j$
  FIR-Filter, HP
  Skizze:



  b) $h[n] = \delta[n+3] - \delta[n+2] + \delta[n+1] - \delta[n]$

  c) $N < -2$

  d) Pol: ∞ (6-fach), Nullstellen: $1, e^{j\pi/4}, e^{j3\pi/4}, -1, e^{j5\pi/4}, e^{7j\pi/4}$; FIR-Filter, BP
  Skizze:



  e) $h_T[n] = \delta[n+6] - \delta[n+4] + \delta[n+2] - \delta[n]$

Gruppe B

- **Beispiel 1**

  a) $y[n] = \alpha(x[n] + y[n-1])$

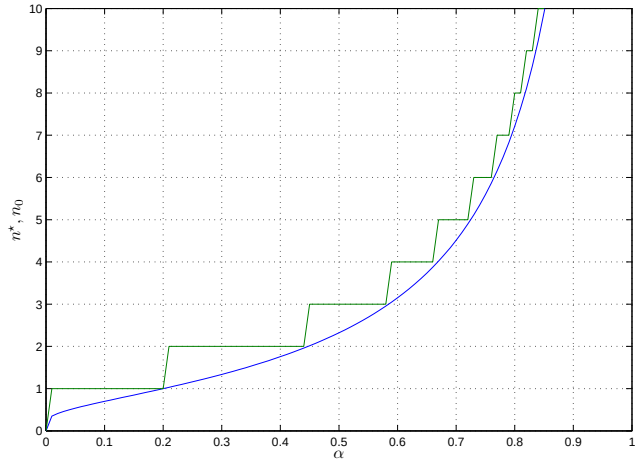
  b) $H(z) = \alpha \frac{z}{z-\alpha}$

  c1) $y[n] = \frac{\alpha}{1-\alpha}(1-\alpha^{n+1})\sigma[n]$

  c2) $\lim_{n\to\infty} y[n] = \frac{\alpha}{1-\alpha}$

  c3) $n_0 = \frac{\log(1/5)}{\log \alpha} - 1$ Rundungsoperation auf nächste ganze Zahl: $n^\star = \lceil n_0 \rceil$

  c4) Skizze:

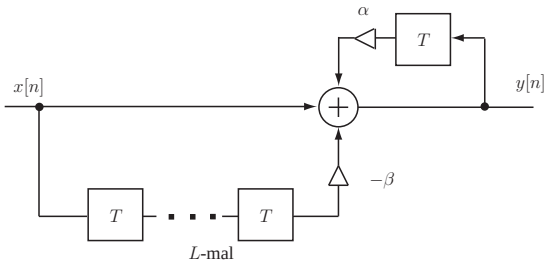


- **Beispiel 2**

  a) $H(z) = \frac{1-\beta z^{-L}}{1-\alpha z^{-1}}$

  b) Pole: $\alpha, 0(L-1)$fach; Nullstellen: $z^L = \frac{1}{\beta} \rightarrow z = |\beta|^{1/L} e^{j(\arg(\beta)+2\pi k)/L}$

  c) Schaltbild:



  d) $h[n] = \alpha^n \sigma[n] + \beta \alpha^{n-L} \sigma[n-L]$

  e) Achtung: $h[n]$ ist unendlich lang, daher ist Abtastung von $H(z)$ falsch! (zeitliches Aliasing)

  Fallunterscheidung: a) $L > N$: $H_a[k] = \sum_{n=0}^{N-1} \alpha^n e^{-j\frac{2\pi}{N}nk} = \frac{1-(-\alpha)^N}{1-\alpha e^{-j\frac{2\pi}{N}k}}$

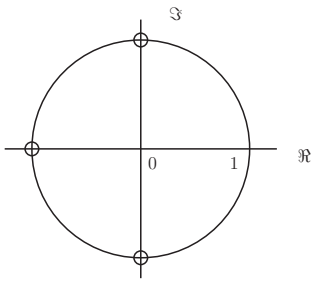
  b) $L < N$: $H_b[k] = H_a[k] - \sum_{n=0}^{N-1} \beta \alpha^{n-L} \sigma[n-L] e^{-j\frac{2\pi}{N}nk}$

  $H_b[k] = H_a[k] - \beta \sum_{n=0}^{N-1-L} \alpha^n \sigma[n-L] e^{-j\frac{2\pi}{N}k(n+L)}$

  $H_b[k] = H_a[k] - \beta \frac{e^{-j\frac{2\pi}{N}kl} - \alpha}{1-\alpha e^{-j\frac{2\pi}{N}k}}$

- **Beispiel 3**

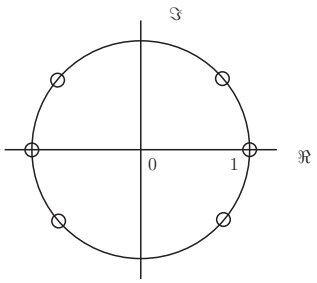
  a) Pol: ∞ (3-fach), Nullstellen: $-1, j, -j$
  FIR-Filter, TP
  Skizze:



  b) $h[n] = \delta[n+3] + \delta[n+2] + \delta[n+1] + \delta[n]$

  c) $N < -2$

  d) Pol: ∞ (6-fach), Nullstellen: $1, e^{j\pi/4}, e^{j3\pi/4}, -1, e^{j5\pi/4}, e^{7j\pi/4}$; FIR-Filter, BP
  Skizze:



  e) $h_T[n] = -\delta[n+6] + \delta[n+4] - \delta[n+2] + \delta[n]$