institute of telecommunications

TECHNISCHE UNIVERSITÄT WIEN Vienna University of Technology

Homework 1

VU Wireless Communications 1, 389.157, SS 2014,
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Important for getting a grade:

- Answer all questions tagged with boxes such as XY short and precise, and state the question number next to the solution.
- Put the homework into the box located at the 1st floor of the *Institute of Telecommunication*, or send it to jkaitovi@nt.tuwien.ac.at.
- Attend the exercise lecture and be prepared to be called to the blackboard for presenting your results.
- In case questions arise, do not hesitate to contact me!

1 Link Budget

Consider a UMTS communication system operating at the carrier frequency $f_c = 2.1\,\text{GHz}$.

3 p [1] Determine the node B sensitivity, the amount of power necessary for the node B to achieve a specific performance level of $\frac{E_b}{N_0} = 1.9\,\text{dB}$ for a user who is using packet switching with the rate of $R_{user} = 128$ kbit/s. The noise in the whole environment is thermal noise, the temperature 20°C. Consider a noise figure $N_f = 3\,\text{dB}$. Fading in this part of the exercise is entirely ignored.

1 p [2] How can you interpret your previous result (result from Question [1]) in the context of a fading channel?

5 p [3] For a mobile user equipment class IV with the maximum output power $P_{\text{UE}} = 125\,\text{mW}$ and the node B from Question [1], calculate the maximum uplink distance from a mobile, located in a building, to a base station, mounted on a rooftop. Take into account that at the first $d_b = 10$ m, the transmit signal can be considered to undergo free-space path loss, while for the rest a path loss exponent of 3.8 is assumed. The penetration loss into buildings is 18 dB. The node B antenna gain is 17 dBi. The effective aperture of the mobile user equipment antenna is $A_e = 10\,\text{cm}^2$.

4 p [4] Imagine now that a UMTS communication system operates at the carrier frequency $f_c = 800\,\text{MHz}$ and that the user equipment antenna gain and node B antenna gain are $G_{\text{ANT}}^{\text{UE}} = 2\,\text{dBi}$ and $G_{\text{ANT}}^{\text{RBS}} = 14\,\text{dBi}$, respectively. Redo the calculations for the uplink budget and find the maximum uplink distance.

1 p [5] How many times is the coverage area increased at 800 MHz compared to the coverage area at 2.1 GHz UMTS system. (Hint: the coverage area can be approximated by $d_{\text{UL}}^2\pi$.)

2 Fading in Mobile Communications Channels

Due to many independent scatterers around the receiver, many copies of the received signal arrive from different angles with unknown random amplitude and phase.

3 p [6] Since we are typically measuring the field strength amplitude $\left(R = \sqrt{I^2 + Q^2}\right)$ rather than its two individual components real (I) and imaginary (Q), which joint distribution in I and Q does the field strength have and what is the marginal distribution of R? Show how the transformation from I and Q to R and θ is done. Assume that the I and Q parts are independent Gaussian distributed with zero mean and standard deviation σ. (Hint: use a conversion from the Cartesian co-ordinate form to the polar co-ordinate form.)

4 p [7] Using MATLAB create $Y = \sum_{k=1}^{6} X_k$, where $X_k \in \mathbb{C}$ are six independent complex random variables, with amplitude $R \sim \mathcal{U}(0, 1)$ and phase $\Theta \sim \mathcal{U}(-\pi, \pi)$. Simulate 10^6 realizations of Y and investigate the distribution of $\mathfrak{Re}\{Y\}$ and $\mathfrak{Im}\{Y\}$ with the `dfittool`.(Hint: use the `rand()` command for uniform distributions.)

2 p [8] Using MATLAB and `dfittool` investigate distributions of the magnitude and the phase of Y from Question [7].

2 p [9] Using the MATLAB function `ksdensity`$(\cdot)$ plot the pdfs of the $\mathfrak{Re}\{Y\}$,$\mathfrak{Im}\{Y\}$, magnitude and the phase of the variable Y from Question [7] and Question [8] and compare your obtained results.

5 p [10] If a magnitude of an electrical field strength $S = |\vec{E}|$ follows a Rayleigh distribution, with probability density function:

$$p_S(s) = \begin{cases} \frac{s}{\alpha} \exp\left(-\frac{s^2}{2\alpha}\right) & ; \quad s \geq 0 \\ 0 & ; \quad \text{else} \end{cases}, \tag{1}$$

analytically derive the distribution of the power $P = S^2$. What is the name of this type of distribution?

The path loss discussed in Section 1 is an average path loss which does not take into account any fading. One way to protect against fading is to transmit with more power.

3 p 11 Assume an application that requires a power outage probability of 5% for the threshold $P_0 = -80\,\text{dBm}$. What value of the average signal power is required for Rayleigh fading?

2 p 12 Assume that in the up-link budget calculations from Section 1 the median power of a Rayleigh fading channel has been considered. This guarantees good reception at 50% of all cases. Using MATLAB calculate the fading margin (in dB) that should be added in order to guarantee a good reception in 99% of the cases. (Hint: use MATLAB function `expinv()`.)

The environment may cause various large-scale variations, shadowing effects, by mountains or large buildings. These variations can be modelled with log-normal distributions. This means that when variations are measured in dB, they follow a Gaussian distribution. Consequently, shadowing effects are usually incorporated into path loss estimates by the addition of a Gaussian random variable, with mean m and standard deviation σ: $\mathcal{N}(m, \sigma)$, were σ is often estimated by empirical measurements.

2 p 13 Assume that the up-link budget calculations are done and that the median value has been considered for the random attenuation due to shadowing. This guarantees good reception at 50% of all cases. Using MATLAB calculate the fading margin (in dB) that should be added in order to guarantee a good reception in 99% of the cases. Assume the standard deviation of the random shadowing of 3 dB. (Hint: Use the MATLAB function `norminv`$(\cdot)$.)

In order to improve the performance of mobile systems, multiple base stations can receive the signal transmitted from a given mobile and combine these multiple signals either by selecting the strongest one or summing the signals together, with some optimized weights. Combining of signals received from multiple base stations is called *macrodiversity*. This typically increases SNR and reduces the effects of shadowing.

1 p 14 Consider a mobile at the midpoint between two base stations in a mobile network with shadowing. Assume that there is no noise in the system and that the powers (in dBm) of the signals received from both base stations are given by:

$$P_{r,1} = P + V_1,$$
$$P_{r,2} = P + V_2,$$

where $V_{1,2} \in \mathcal{N}(m_V, \sigma_V^2)$ are random variables. Interpret the terms P, V_1, V_2 in $P_{r,1}$ and $P_{r,2}$.

2 p

[15] If V_1 and V_2 from Question [14] are independent and if an outage in the presence of macrodiversity occurs when both $P_{r,1}$ and $P_{r,2}$ fall below a threshold T, show that the outage probability is given by:

$$P_{\text{out}} = \left[Q\left(\frac{\Delta}{\sigma}\right) \right]^2$$

where $Q(z) \triangleq p(x > z)$ is defined as the probability that a Gaussian random variable x with zero mean and unit variance is larger than z, and $\Delta = P - T$ is the fading margin at the position of the mobile.