Homework 2

VU Wireless Communications 1, 389.157, SS 2014,
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Important for getting a grade:

- Answer all questions tagged with boxes such as [XY] short and precise, and state the question number next to the solution.
- Put the homework into the box located at the 1st floor of the *Institute of Telecommunication*, or send it to jkaitovi@nt.tuwien.ac.at.
- Attend the exercise lecture and be prepared to be called to the blackboard for presenting your results.
- In case questions arise, do not hesitate to contact me!

1 Linear Time-Variant Systems

2 p [1] Consider a wireless LAN operating at a subway station. The transmitter and receiver have a LOS path between them with attenuation α_0, phase ϕ_0 and delay τ_0. Every T_0 seconds a train comes through the station, creating an additional reflected signal path in addition to the LOS path with attenuation α_1, phase ϕ_1 and delay τ_1. Find the time-variant impulse response $h(\tau, t)$ of this channel.

3 p [2] Analytically calculate the *time-variant transfer function* $T(t, f)$ of the time-variant impulse response from Question [1].

3 p [3] Analytically calculate the *Doppler-variant impulse response* $S(\nu, \tau)$ of the time-variant impulse response from Question [1]. (Hint: use the Poisson summation formula.)

3 p [4] Analytically calculate the *Doppler-variant transfer function* $H(\nu, f)$ of the time-variant impulse response from Question [1] . (Hint: use the Poisson summation formula.)

2 Characterization of Time-variant Channels

A power delay profile is given by:

$$P_h(\tau) = \alpha P_0 \exp\left(-\frac{\tau - \tau_0}{T_0}\right) u(\tau - \tau_0), \quad (1)$$

where P_0 is a constant power, T_0 is a constant time, τ is a delay and $u(\cdot)$ is the Heaviside step function. Furthermore, α is an attenuation and τ_0 is a constant time delay.

2 p [5] Using MATLAB draw the power delay profile. Under which name is this PDP known in literature? (Hint: Just for the drawing of the constants, take the following values: $\alpha = \frac{1}{2}$, $P_0 = 1$, $\tau_0 = 0$ and $T_0 = 1$.)

A Doppler spectral density $P_B(\nu)$ is given by:

$$P_B(\nu) = \begin{cases} \frac{\beta P_0}{\pi\sqrt{4\nu_{\max}^2 - \nu^2}} & ; \text{if } |\nu| < 2\nu_{\max} \\ 0 & ; \text{else} \end{cases} \quad (2)$$

where, β is an attenuation, P_0 is a constant power, $\nu_{\max}$ is the maximum Doppler frequency and ν is the frequency.

2 p [6] Using MATLAB draw the Doppler spectral density. Under which name is this spectrum known in literature? (Hint: Just for the drawing of the constants, take the following values: $\beta = 2$, $P_0 = 1$ and $\nu_{\max} = \frac{1}{2}$.)

2 p [7] Assume that ν and τ are uncorrelated and formulate the expression for the *scattering function* $P_S(\nu, \tau)$.

3 Moments of the Power Delay Profile

The power delay profile is a one-dimensional function. This detailed information can be condensed in a single parameter, capturing the most important channel property.

2 p [8] Starting from the power delay profile given in Section 2 compute the *mean power* P_m analytically.

3 p [9] Compute analytically the *mean delay* T_m.

3 p [10] Compute analytically the *rms delay spread* S_τ.

In a frequency-selective channel, different frequencies fade differently. The *coherence bandwidth* defines the bandwidth in which the fading is roughly equal (the fading across the entire signal bandwidth is highly correlated).

1 p [11] Use the *Fleury inequality* to find a bound on the *coherence bandwidth* B_{coh} (in Hz). Use S_τ from Question [10].

4 Moments of the Doppler Spectra

3 p [12] Starting from the Doppler spectral density $P_B(\nu)$ given in Section 2 compute the *mean power* P_B, m analytically.

3 p [13] Compute analytically the *mean Doppler shift* ν_m.

4 p [14] Find the *rms Doppler spread* S_ν. (Hint: you may use a CAS[1] for computing integrals (e.g. `http://integrals.wolfram.com`).)

1 p [15] Use the *Fleury inequality* to find a bound on the *coherence time* T_{coh}. Use S_ν from Question [14].

2 p [16] Consider a system operating at the carrier frequency $f_c = 2.1\,\text{GHz}$. Find the maximum vehicular speed $v_{\max}$ so that one pilot symbol per frame, with a duration of $T_f = 512\,\mu s$, is enough for channel tracking. Assume that one pilot symbol is enough to estimate the channel correctly for a duration of T_{coh}. Use T_{coh} from Question [15].

[1] CAS (computer algebra system) is a software program that facilitates symbolic mathematics.