

Homework 3

VU Wireless Communications 1, 389.157, SS 2014,
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Important for getting a grade:

- Answer all questions tagged with boxes such as \boxed{XY} short and precise, and state the question number next to the solution.
- Put the homework into the box located at the 1st floor of the *Institute of Telecommunication*, or send it to jkaitovi@nt.tuwien.ac.at.
- Attend the exercise lecture and be prepared to be called to the blackboard for presenting your results.
- In case questions arise, do not hesitate to contact me!

1 Digital Modulation for Wireless Transmissions

- 1 p $\boxed{1}$ Consider the modulation scheme shown in Figure 1. Determine the minimum distance between two symbols d_{\min} , the average energy per symbol E_s and the average energy per transmitted bit E_b .

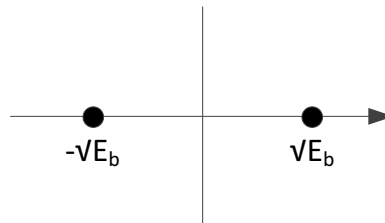


Figure 1: BPSK signal space diagram.

- 3 p $\boxed{2}$ Compute the symbol error probability P_s in AWGN as a function of energy per bit E_b for BPSK from Question [1]. (Hint: Only noise that is in the in-phase component matters $N_I \sim \mathcal{N}(0, \frac{N_0}{2})$.)

When the BPSK signal is transmitted over Rayleigh fading channels, the received signal is:

$$r = \alpha \sqrt{E_b} s_k + n, \quad (1)$$

where α denotes the fading amplitude and is Rayleigh distributed:

$$f(\alpha) = \frac{\alpha}{\sigma_\alpha^2} \exp\left(-\frac{\alpha^2}{2\sigma_\alpha^2}\right), \quad \alpha \geq 0, \quad (2)$$

and $\sigma_\alpha^2 = E[\alpha^2]$.

- 3 p [3] Starting from the pdf of α given in Equation (2), find the pdf of γ where $\gamma = \frac{\alpha^2 E_b}{N_0} = \frac{\alpha^2}{\sigma_\alpha^2} \gamma_c$.
- 5 p [4] Analytically derive the BEP of BPSK over flat Rayleigh fading channels. (Hint: Use the derivation from Question [3] and see Appendix A.)
- 5 p [5] Using the Quadratic Form Gaussian Variable (QFGB) method compute the BEP of BPSK over flat Rayleigh fading channels. Check your results by comparing with the result of Question [4].
- 2 p [6] Find the minimum average SNR per bit required in order to achieve $P_b < 10^{-2}$.
- 2 p [7] Using MATLABplot BEP vs. SNR per bit performance of the BPSK system when communicating over AWGN and Rayleigh fading channels.
- 2 p [8] Calculate the margin (in dB) necessary to be added when working in a fading environment in order keep the same performance (a)BEP= 10^{-2} and (b)BEP= 10^{-3} .
- 3 p [9] Determine the required $\gamma_b = \frac{E_b}{N_0}$ for BPSK in slow Rayleigh fading such that 75% of the time (or in space), $P_b(\gamma_b) < 3 \cdot 10^{-3}$. (Hint: use $P_b = Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right)$)

- 4 p [10] Perform a Monte Carlo simulation for a BPSK communication system. The block diagram of the system is shown in Figure 2.

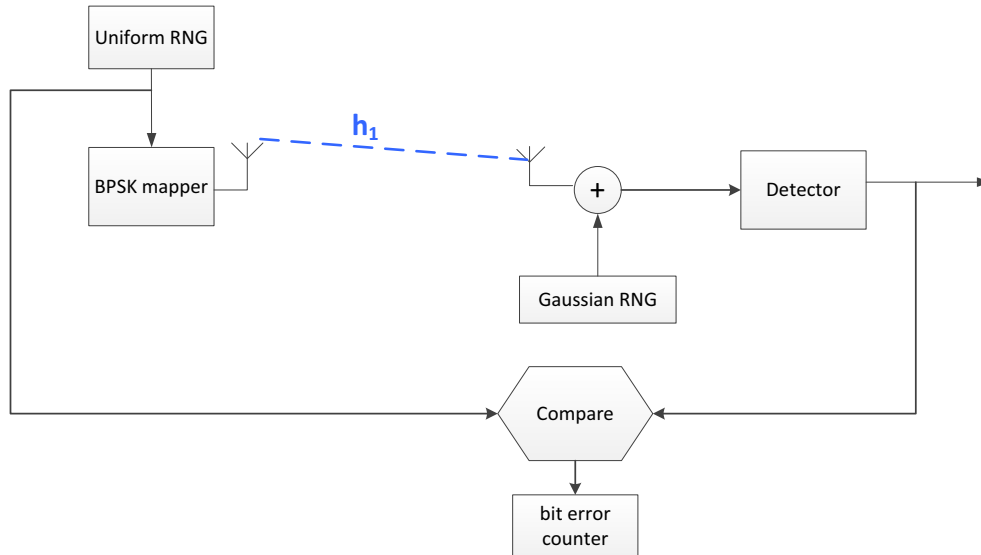


Figure 2: Block diagram of a BPSK communication system.

Simulate $N = 10^7$ bits and plot BER vs. SNR per bit in MATLAB. The noise in the system is modelled by a Gaussian RNG¹. Evaluate your results by comparing with the theoretically calculated performance of a system.

- 6 p [11] Consider the system described in Question [10]. In addition to the previous, now there is a channel between a transmitter and a receiver and its setup is modelled by a Rayleigh distributed block fading channel. Assume perfect channel knowledge at the receiver. Simulate $N = 100$ bits and plot BER vs. SNR per bit in MATLAB. Around each point in the graph (around BER values) plot a confidence interval that contains 95% of results. Repeat the simulation for each point suitable number of times so that the confidence interval is not too large (i.e., less than 0.05). Evaluate your results by comparing with the theoretically calculated performance of a system.

¹RNG is a random number generator.

A Appendix

Changing the integration order

The order of the integration can be changed from:

$$\int_0^8 \int_{\sqrt[3]{y}}^2 f(x, y) dx dy, \quad (3)$$

to:

$$\int_0^2 \int_0^{x^3} f(x, y) dy dx. \quad (4)$$

See Figure 3.

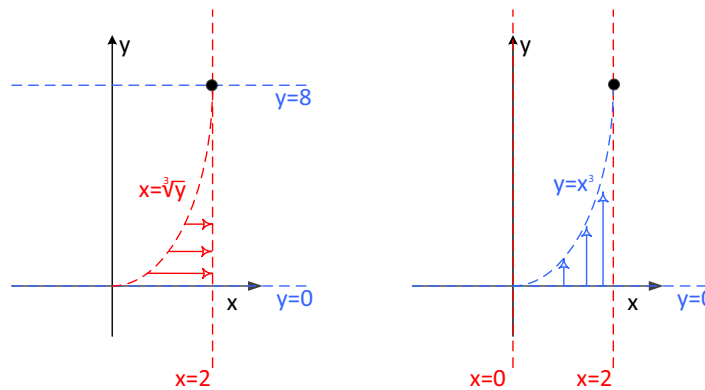


Figure 3: The domain of integration and limits.