institute of telecommunications

TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

Homework 4

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Important for getting a grade:

- Answer all questions tagged with boxes such as $\boxed{\text{XY}}$ short and precise, and state the question number next to the solution.
- Put the homework into the box located at the 1st floor of the *Institute of Telecommunication*, or send it to jkaitovi@nt.tuwien.ac.at.
- Attend the exercise lecture and be prepared to be called to the blackboard for presenting your results.
- In case questions arise, do not hesitate to contact me!

1 Digital communications through fading multipath channels

Suppose that the binary signal $\pm s_k(t)$ is transmitted over a block fading channel and the received signal is:

$$r_i(t) = \pm h s_k(t) + n(t), \qquad 0 \leq t \leq T, \tag{1}$$

where $n(t)$ is a zero-mean white Gaussian noise with autocorrelation function

$$R_{n,n}(\tau) = N_0 \delta(\tau). \tag{2}$$

The energy in the transmitted signal is $E_s = \frac{1}{2}\int_0^T |s_k(t)|^2 dt$. The channel gain h is specified by the probability density function:

$$p(h) = 0.2\delta(h) + 0.8\delta(h-2). \tag{3}$$

2 p $\boxed{1}$ Determine the average probability of error P_e for the demodulator that employs a filter matched to s_k.

1 p $\boxed{2}$ What value does P_e approach as $\frac{E_s}{N_0}$ approaches infinity.

3 p $\boxed{3}$ Suppose that the same signal is transmitted on two statistically independent fading channels with gains h_1 and h_2, where:

$$p(h_l) = 0.2\delta(h_l) + 0.8\delta(h_l - 2), \qquad l = 1, 2. \tag{4}$$

The noise on the two channels is statistically independent and identically distributed. The demodulator employs a matched filter for each signal and simply adds the two filter outputs to form the decision variable. Determine the average P_e.

1 p $\boxed{4}$ For the case from Question [3] what value does P_e approach as $\frac{E_s}{N_0}$ approaches infinity.

When the BPSK signal is transmitted over Nakagami-m fading channels, the received signal is:

$$r = h\sqrt{E_b}s_k + n, \tag{5}$$

where h denotes the fading amplitude, Nakagami-m distributed:

$$f(h) = \frac{2h^{2m-1}}{\Gamma(m)} \left(\frac{m}{\sigma_h^2}\right)^m \exp\left(-\frac{mh^2}{\sigma_h^2}\right), \qquad h \geq 0, \tag{6}$$

where $\Gamma(\cdot)$ is the gamma function, $\sigma_h^2 = E[h^2]$ and m is the Nakagami-m fading parameter.

3 p $\boxed{5}$ Starting from the pdf of h given in Equation (6), find the pdf of γ where $\gamma = \frac{h^2 E_b}{N_0}$.

6 p $\boxed{6}$ Analytically derive the BEP of BPSK over Nakagami-m channel.
Hints:

1. assume that m is a positive integer
2. use the derivation from Question [5]
3. $Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\phi}\right) d\phi$,
4. Laplace transform $\int_0^\infty x^\nu e^{-sx} dx = \frac{\Gamma(\nu+1)}{s^{\nu+1}}$

5.

$$\int_0^{\frac{\pi}{2}} \left(\frac{\sin^2 \phi}{\sin^2 \phi + c} \right)^m d\phi = \pi \left[P(c) \right]^m \sum_{j=0}^{m-1} \left(\begin{array}{c} m-1+j \\ j \end{array} \right) (1-P(c))^j \,,$$

where $P(c) = \frac{1}{2}\left(1 - \sqrt{\frac{c}{1+c}}\right)$.

3 p $\boxed{7}$ Using MATLAB plot BEP vs. SNR per bit of the BPSK system when communicating over Nakagami-m channel for $m = [1, 2, 5, 20]$ and compare with the BEP of BPSK over an AWGN channel.

3 p $\boxed{8}$ Calculate the margin (in dB) necessary to be added to keep the same performance BEP=10^{-3} when working in a fading environment (a) m=1, (b) m=2 and (c) m=20.

2 Diversity

Consider a digital communication system that uses two transmitting antennas and one receiving antenna. The two transmitting antennas are sufficiently separated so as to provide dual spatial diversity in the transmission of the signal. The transmission scheme is as follows: If s_1 and s_2 represents a pair of symbols from either a one-dimensional or a two-dimensional signal constellation, which are to be transmitted by the two antennas, the signal from the first antenna over two signal intervals is (s_1, s_2) and from the second antenna the transmitted signal is $(s_2^*, -s_1^*)$. The signal received by the single receiving antenna over the two signal intervals is:

$$\begin{aligned} r_1 &= h_1 s_1 + h_2 s_2 + n_1 \\ r_2 &= h_1 s_2^* - h_2 s_1^* + n_2, \end{aligned} \tag{7}$$

here, (h_1, h_2) represent the complex-valued channel path gains, which may be assumed to be zero-mean, complex Gaussian with unit variance and statistically independent. The channel path gains (h_1, h_2) are assumed to be constant over the two signal intervals and known to the receiver. The terms (n_1, n_2) represent additive white Gaussian noise terms that have zero-mean and variance σ_n^2 and is uncorrelated.

4 p $\boxed{9}$ Show how to recover the transmitted symbols (s_1, s_2) from (r_1, r_2) and achieve dual diversity reception by computing the SNR for a fixed pair (h_1, h_2).

3 p $\boxed{10}$ If the energy in the pair (s_1, s_2) is (E_s, E_s) and the modulation is BPSK determine the probability of error. (Hint: For approximation use Taylor series of $\sqrt{1+x} = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{(1-2n)(n!)^2 4^n} x^n$.)

3 p $\boxed{11}$ Repeat the Question [10] if the modulation is QPSK.