institute of telecommunications

TECHNISCHE UNIVERSITÄT WIEN
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Homework 8

VU Wireless Communications 1, 389.157, SS 2014,
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Important for getting a grade:

- Answer all questions tagged with boxes such as XY short and precise, and state the question number next to the solution.
- Put the homework into the box located at the 1st floor of the *Institute of Telecommunication*, or send it to veronika.shivaldova@nt.tuwien.ac.at.
- Attend the exercise lecture and be prepared to be called to the blackboard for presenting your results.
- In case questions arise, do not hesitate to contact me!

1 Transmit Power Control and SIR Balancing

Assume that you have a noiseless ad-hoc network consisting of 3 transmitter - receiver pairs. The channel gain between the transmitter TX_j and receiver RX_i decreases with the 2_d power of the distance. The transmitters and the receivers have the following coordinates:

- TX_1 (10, 15), RX_1(20, 25)
- TX_2 (15, 20), RX_2(15, 30)
- TX_3 (20, 35), RX_3(25, 35)

Signal arriving from transmitter TX_j to any receiver RX_i with $i \neq j$ is considered as interference.

5 p 1 Determine if the target SIR $\gamma_0 = 5\,\text{dB}$ is achievable.

5 p 2 Using the SIR balancing principle find the largest SIR that can be achieved in all three receiving nodes?

5 p 3 How should the powers of TX_2 and TX_3 be related to the power of TX_1, to achieve the largest minimum SIR calculated in Question 2?

5 p 4 How large is the SIR (in dB) obtained at each receiver, if the power of all three transmitters is exactly the same?

5 p 5 Assume that the transmit power is chosen such, that the received power for all three receivers is exactly the same. How large are the corresponding SIR values (in dB)?

2 Properties of MIMO Channels

Consider a MIMO channel with $M_r \times M_t$ matrix $\mathbf{H}$ of channel coefficients, that is known to both the transmitter and the receiver.

5 p 6 Show that matrix $\mathbf{HH}^H$ is Hermitian. What does that reveal about the eigenvalues of $\mathbf{HH}^H$?

5 p 7 Show that $\mathbf{HH}^H$ is positiv semidefinite.

5 p 8 Show that $\mathbf{I_M} + \mathbf{HH}^H$ is Hermitian positive definite.

The multiplexing gain of the systems with multiple transmit and receive antennas results from the fact that the MIMO channel can be decomposed into a number of parallel independent channels. By multiplexing independent data onto these independent channels we get an increase in data rate in comparison to systems with just one antenna at the transmitter and receiver. This data rate increase is called multiplexing gain.

5 p 9 What is the maximum multiplexing gain of a system with the following $M_r \times M_t$ matrix of channel coefficients:

$$\mathbf{H} = \begin{bmatrix} 0.7 & 0.6 & 0.2 & 0.4 \\ 0.1 & 0.5 & 0.9 & 0.2 \\ 0.3 & 0.6 & 0.9 & 0.1 \end{bmatrix}$$

5 p 10 How large are the channel gains of the independent parallel SISO channels?

3 Alamouti's Space-Time Coding

This exercise deals with Alamouti space-time coding that involves transmission of multiple redundant copies of data to compensate for fading effects introduced by a wireless channel. For so-called space-time block coded transmission the transmit data stream is encoded in blocks, which are distributed in space (several antennas) and time (several time instances).

- At first generate a block consisting of 20 equally probable, random 4QAM symbols, such that possible values are $1 + j$, $1 - j$, $-1 + j$ and $-1 - j$. Normalize the constellation to unit transmit power.
- Perform Alamouti space-time block coding by taking two consecutive symbols $\mathbf{s} = \{s_1, s_2\}^T$, and transmitting the sequence $\{s_1, s_2^*\}^T$ on the first and $\{s_2, -s_1^*\}^T$ on the second antenna.
- Now simulate the transmission over a multiplicative fading channel with an additive white Gaussian noise as shown in Fig, 1. For this assume i.i.d. channel coefficients $h_1, h_2 \sim \mathcal{CN}(0, 1)$ and zero-mean complex Gaussian noise. Assume block fading scenario, i.e., channel coefficients h_1 and h_2 remain constant during transmission of one block of symbols.

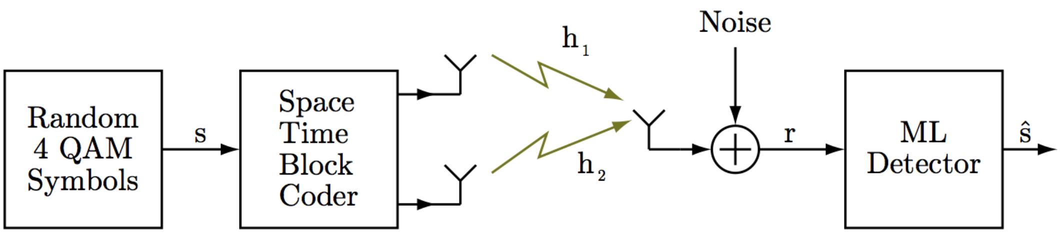

Figure 1: Alamouti 2x1 transmission.

- The receive vector $\mathbf{r}$ is then formed by the two consecutive data samples received in the following way (note the conjugation of $\mathbf{r}_2$):

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2^* \end{bmatrix}$$

- In order to perform the decoding of the received samples for a single receive antenna assume that the receiver has perfect channel knowledge. To this end, we introduce a virtual channel matrix

$$\mathbf{H_v} = \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix}$$

- Finally use zero forcing receiver to estimate the transmitted symbols $\hat{\mathbf{s}}$ as follows:

$$\hat{\mathbf{s}} = (\mathbf{H}_v^H \mathbf{H_v})^{-1} \mathbf{H}_v^H \mathbf{r}$$

5 p [11] Show analytically that in the absence of noise, $\hat{\mathbf{s}}$ is equal to $\mathbf{s}$ for arbitrary channel coefficients $h_1 \neq 0$ and $h_2 \neq 0$.

15 p [12] Use average channel SNR values in range [-10 dB, 20 dB] with step of 1 dB to simulate the transmission. For each transmitted block calculate BER and estimate SNR at the receiver. Repeat the same procedure for sufficient number of channel realization to obtain an average BER and SNR. Using MATLAB function `semilogy` plot the resulting BER over SNR curve.

10 p [13] Show by MATLAB simulation that the BER over SNR performance of 2x1 Alamouti transmission matches the theoretical result obtained by calling the MATLAB function `xmBER_Div(SNR_dB, 1)`, **if antenna 1 is broken, i.e.,** $\mathbf{h_1 = 0}$.

5 p [14] Interpret the result obtained in question 13 and show analytically that if antenna 1 is broken, the transmission behaves like a SISO transmission in a Rayleigh fading scenario.

10 p [15] Show by MATLAB simulation that the BER over SNR performance of a 2x1 Alamouti transmission matches the theoretical result given by calling the MATLAB function `xmBER_Div(SNR_dB, 1)`, **if the transmission paths originating in antenna 1 and antenna 2 are perfectly correlated , i.e.,** $\mathbf{h_1 = h_2}$.

5 p [16] Interpret the result obtained in question 15 and show analytically that if the transmission paths originating in antenna 1 and antenna 2 are perfectly correlated, the transmission behaves like a SISO transmission in a Rayleigh fading scenario.