institute of telecommunications

TECHNISCHE UNIVERSITÄT WIEN Vienna University of Technology

Homework 1

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Important for getting a grade:

- Answer all questions tagged with boxes such as [XY] short and precise, and state the question number next to the solution.
- Put the homework into the box located at the 1st floor of the *Institute of Telecommunication*, or send it to jkaitovi@nt.tuwien.ac.at.
- Attend the exercise lecture and be prepared to be called to the blackboard for presenting your results.
- In case questions arise, do not hesitate to contact me!

1 Link Budget

Consider a transmitter with a transmit power of 50 W.

1 p [1] Express the transmit power in units of:

a) dBm

b) dBW.

3 p [2] If 50 W is applied to an isotropical antenna with a 2.1 GHz carrier frequency. Find the received power in dBm at a free space distance of 100 m from the antenna. Assume unity gain for the receiver antenna.

1 p [3] Find the receiver power in dBm at a free space distance of 10 km from the transmit antenna.

2 p [4] If the affective aperture of the mobile user antenna is $A_e = 10\,\text{cm}^2$, recalculate the received power.

1 p [5] Explain the advantages and disadvantages of the two-ray ground reflection model in the path loss analysis.

2 p [6] Can a two-ray model be applied in the following cases? Explain why or why not:

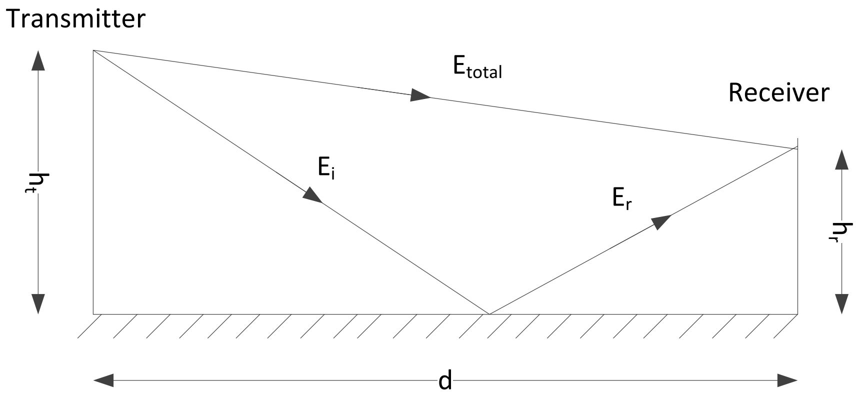

Figure 1: Two-ray ground reflection model

a) $h_t = 35\,m$, $h_r = 13\,m$ and $d = 250\,m$

b) $h_t = 3\,m$, $h_r = 1.5\,m$ and $d = 450\,m$

2 Fading

5 p [7] If the magnitude of an electrical field strength $S = |\vec{E}|$ follows a Rayleigh distribution with probability density function:

$$p_S(s) = \begin{cases} \frac{s}{\alpha} \exp\left(-\frac{s^2}{2\alpha}\right) & ; \quad s \geq 0 \\ 0 & ; \quad \text{else} \end{cases}, \tag{1}$$

analytically derive the distribution of power $P = S^2$. What is the name of this type of distribution?

The path loss discussed previously is an average path loss which does not take into account any fading. One way to protect against fading is to transmit with more power.

3 p [8] Assume an application that requires a power outage probability of 25% for the threshold $P_0 = -50\,\text{dBm}$. What value of the average signal power is required for Rayleigh fading?

2 p [9] Assume that in the budget calculations of Question [2] the median power of a Rayleigh fading channel has been considered. This guarantees good reception at 50% of all cases. Using MATLAB calculate the fading margin (in dB) that should be added in order to guarantee a good reception in 90% of the cases. (Hint: use MATLAB function `expinv()`.)

3 Shadowing and Coverage

The probability that the received signal level will exceed a certain value γ can be calculated from a cumulative density function as:

$$P_r\left[P_r(d) > \gamma\right] = Q\left(\frac{\gamma - \overline{P_r(d)}}{\sigma}\right), \tag{2}$$

where:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{u^2}{2}\right) du, \tag{3}$$

and σ is the standard deviation of $P_r(d)$ and $\overline{P_r(d)}$ denotes its mean.

5 p **10** Consider a transmitter providing 15 W to an antenna with 6 dB gain. The receiver antenna has a gain of 3 dB and the receiver bandwidth is 30 kHz. If the receiver system noise figure is 8 dB and the carrier frequency is 2.1 GHz, find the maximum transmitter-receiver separation that will ensure that an SNR of 20 dB is provided for 95% of users. Assume $n = 4$, $\sigma = 8$ dB and $d_0 = 1$ km. (Hint: use the MATLAB function `qfuncinv`(·).)

3 p **11** Four measurements of the received power were taken at distances of 100 m, 200 m, 1 km, and 3 km from a transmitter. These measured values are given in Table 1. It is assumed that the path loss for these measurements follows the model:

$$\mathrm{PL(d)\,[dB]} = \overline{\mathrm{PL(d)}} + X_\sigma = \overline{\mathrm{PL(d_0)}} + 10 \cdot n \cdot \log\left(\frac{d}{d_0}\right) + X_\sigma, \tag{4}$$

where $d_0 = 100$ m. Find the minimum mean square error (MMSE) estimate for the path loss exponent n.

Table 1: Measured values.

distance from transmitter	received power
100 m	0 dBm
200 m	-20 dBm
1 000 m	-35 dBm
3 000 m	-70 dBm

2 p | 12 | Calculate the standard deviation about the mean value for the problem of Question [11].

2 p | 13 | Using the resulting model from Question [11] and Question [12], estimate the median received power at $d = 2\,\mathrm{km}$.

2 p | 14 | Predict the likelihood that the received signal level at 2 km will be greater than -60 dBm. (Hint: use the MATLAB function `qfunc(·)`.)

3 p | 15 | Assume that field measurements of local average signal strength were made inside a building, and post processing revealed that the measured data fit a distant-dependant mean power law model having a log-normal distribution about the mean. Assume the mean power law was found to be $\overline{P_r(d)} \propto d^{-3.5}$. If a signal of 1 mW was received at $d_0 = 1\,\mathrm{m}$ from the transmitter, and at a distance of 10 m, 10% of the measurements were stronger than -25 dBm, define the standard deviation, σ, for the path loss model at $d = 10\,\mathrm{m}$. (Hint: use the MATLAB function `qfuncinv(·)`.)